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PROCEEDING

INTERNATIONAL CONFERENCE ON STATISTICS, MATHEMATICS, TEACHING, AND RESEARCH

ICSMTR 2015

Increasing Statistical and Mathematical Literacy through High Quality Teaching and Research

October 9-10, 2015
Makassar, South Sulawesi, Indonesia

STATISTICS DEPARTMENT AND MATHEMATICS DEPARTMENT
STATE UNIVERSITY OF MAKASSAR
INDONESIA

CONFERENCE PROCEEDING

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WELCOME SPEECH

Forewords from the Head of Committee

Bismillahirrahmanirrahim
Assalamu'alaikum Warahmatullahi Wabarakatuh

First, I want to give our welcome to all the delegates, speakers, and participants coming today. Welcome to the State University of Makassar, UNM.

This International Conference on Statistics, Mathematics, Teaching, and Research (ICSMTR) 2015 is primarily organized by Statistics Department and Mathematics Department, Faculty of Mathematics and Sciences, State University of Makassar. It is conducted in two days from 9th to 10th October 2015. It involves one keynote speaker, Governor of South Sulawesi, eight invited speakers, and approximately 80 parallel speakers. Besides, this conference also invites delegates from twelve LPTKs (Institute of Teacher Education) to conduct a scientific meeting reviewing KKNI for Mathematics Education curriculum in higher education.

Ladies and gentlemen, as I previously said, the conference proudly invites eight invited speakers coming from several countries. Therefore, on behalf of the committee members, I would like to express my sincere thanks to the invited speakers, specifically:

1. Professor Kerrie Mengersen (Queensland University of Technology, Australia)
2. Professor Shigehiko Kanaya (Nara Institute of Science and Technology, Japan)
3. Professor Ahmad A. Bahnassy (Faculty of Medicine, King Fahd Medical City, Saudi Arabia)
4. Professor I Gusti Ngurah Agung (State University of Makassar, Indonesia)
5. Professor Hamzah Upu (State University of Makassar, Indonesia)
6. Professor Muhammad Arif Tiro (State University of Makassar, Indonesia)
7. Professor Mohd. Salmi Md Noorani (Universiti Kebangsaan Malaysia, Malaysia)
8. Dr. Darfiana Nur (Flinders University, Australia)

Next, it is my privilege to thank all organizing committee members for their contributions to the success of this event. I would like also to apologize for all of you if there are some inconvenience during this conference.

Finally, I would like to thank to the speakers and participants. I wish you all have two fruitful days in Makassar.

Thank you very much for the attention.

Wassalamu'alaikum Warahmatullahi Wabarakatuh

Suwardi Annas, Ph.D.
Head of Committee

Forewords from the Dean of Mathematics and Sciences Faculty,

State University of Makassar

Bismillahirrahmanirrahim
Assalamu'alaikum Warahmatullahi Wabarakatuh

Alhamdulillah, all praises be to the Almighty God, Allah *subhanahu wata'ala*.

I would like to say that I welcome and highly appreciate any attempts of both the Statistics Department and Mathematics Department to organize this International Conference on Statistics, Mathematics, Teaching, and Research in the State University of Makassar. I do hope that this conference would be a great chance for you as researchers or scholars in enhancing your research quality within a framework of evolving sciences. May Allah *subhanahu wata'ala* opens our mind, widens our view, strengthens our soul, and blesses our conference that it will be useful as we are hoping.

At last, as the Dean of the Faculty of Mathematics and Natural Sciences, State University of Makassar (FMIPA UNM), I am sure that there are some weaknesses and mistakes in performing this conference. I therefore do apologize to you and may Allah *subhanahu wata'ala* forgive all of us.

Wassalamu'alaikum Warahmatullahi Wabarakatuh

Professor Abdul Rahman

Dean of Faculty of Mathematics and Sciences

State University of Makassar

Forewords from Rector of UNM

Bismillahirrahmanirrahim
Assalamu'alaikum Warahmatullahi Wabarakatuh

Your respectable, the high officials of State University of Makassar, the committee, the speakers, and the participants of conference.

It gives me great pleasure to extend to you all a very warm welcome, especially to our keynote speakers who have accepted our invitation to convene the conference. ICSMTR is one of our educational activities that covers a wide range of very interesting items relating to statistics, mathematics, teaching and research.

By taking participation of this conference, it is highly expected to all of us to share our research findings to society and continuously develop new ideas and knowledge. Those things are two significant steps in improving the quality of nations around the world, increasing our familiarity to each other, and even avoiding underdevelopment.

Furthermore, I would like to take this opportunity to express my heartfelt gratitude to all organizing committee especially for Statistics Department and Mathematics Department of Faculty Mathematics and Natural Sciences that primarily hosts this conference.

Finally, this is a great time for me to declare the official opening of the International Conference on Statistics, Mathematics, Teaching, and Research (ICSMTR) 2015.

I wish you a very enjoyable stay in Makassar
I warmly welcome you again, as in Makassar, we say "*salamakki battu ri mangkasara*"

Wassalamu'alaikum Warahmatullahi Wabarakatuh.

Prof. Dr. H. Arismunandar, M.Pd.
Rector of State University of Makassar

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ANALYSIS OF SUSCEPTIBLE, INFECTED, RECOVERED, SUSCEPTIBLE (SIRS) MODEL FOR SPREAD OF THE ACUTE RESPIRATORY TRACT INFECTIONS (ARI) DISEASE

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ABSTRACT

ARI is an acute inflammation of the upper and lower respiratory tract caused by infection with microorganisms or bacteria, viruses, and rickets, without or with inflammation of the lung parenchyma, such that it can cause a spectrum of illnesses ranging from asymptomatic disease or mild infection to a deadly disease, depending on the causative pathogen. Therefore, it is necessary to analyze scientifically acceptable to the events of the spread of respiratory diseases. One of them can be seen in the form of a mathematical model, that is, Susceptible, Infected, Recovered, Susceptible (SIRS) model. Determination of equilibrium points and eigenvalue equation, as well as basic reproduction numbers (R_0) were done to analyze the stability of the model. From the analysis of model, it found that the free disease equilibrium point, model is asymptotically stable when R_0≤1, while for the endemic equilibrium point, model is asymptotically stable when R_0>1. It indicates that, the Bah Jambi district is free from the ARI disease. Collecting real data and simulation of the spread of ARI using the fourth order Runge-Kutta (RK4) was done to validity of model.

Keyword: ARI Desease, SIRS Model

1. INTRODUCTION

ARI is an acute inflammation of the upper and lower respiratory tract caused by infection with microorganisms or bacteria, viruses, and rickets, without or with inflammation of the lung parenchyma (Alsagaff dan Mukty, 2009). This disease is a contagious disease and can cause a spectrum of illnesses ranging from asymptomatic disease or mild infection to a deadly disease, depending on the causative pathogen, environmental factors and host factors. ARI caused by an infectious agent transmitted from human to human. Symptoms include fever, cough, and often sore throat, coryza (runny nose), shortness of breath, chills, or difficulty breathing (WHO, 2007). Infectious diseases collected in a Basic Health Research (Riskesdas) in 2013 shows that the period prevalence ARI based diagnosis of health workers and complaints population in Indonesia is 25%. ARI disease prevalence values are higher than other airborne infectious diseases, namely pneumonia (4.5%) and pulmonary TB (0.4%) (Riskesdas, 2013). And periods of cold cough disease that is part ARI illness in children under five in Indonesia is estimated to 3-6 times per year (an average of 4 times per year), meaning an average toddler having attacks of cough and cold as much as 3-6 times a year (Widoyono, 2005).

Based on data from Maraja Java Health Center in 2013-2014 ARI is a disease first level of 10 cases of diseases in the Maraja Bah Jambi Java District. It is necessary to investigate how the spread of the respiratory disease. Authors conducted a study of transmission of respiratory diseases at the health center Maraja Java through mathematical models. A mathematical model that can be applied to determine the amount of the spread of respiratory diseases in the area..

The mathematical model used is a model susceptible epidemic, Infected, recovered, susceptible (SIRS). SIRS Model is a model of the spread of diseases that divide the population into three classes, individuals susceptible (susceptible), a class of individuals infected (Infected) and individual classes recovered (Recovered). SIRS epidemic models is an extension of the classical model of SIR that has been put forward by Hetcote in 1976 and 198 (Rohmah dan Kusumawinahyu, 2014). The purpose of this study are: (1) Determine the model of SIRS in the spread of the disease in the Maraja Bah Jambi Java District. (2) Determine the stability analysis on the spread of ARI. (3) Knowing the Maraja Bah Jambi Java District dangerous or not the ARI disease. (4) Knowing the numerical simulations on the spread of ARI using Runge-Kutta method of order 4.

2. MATHEMATICAL MODEL

The establishment of a mathematical model is constrained by a number of assumptions. The assumptions used in the model of the spread of respiratory diseases as follows: (1) Model SIRS illustrates that individuals susceptible to disease become infected individual illness, healing, immunity while against the disease, after it because immunity is disappearing then the individual reentry in vulnerable populations (susceptible). (2) The rate of births occurring in the population assumed to be equal to the rate of death, so that the total population is assumed to remain constant and do not pay attention to the incubation period (time of transmission). (3) The spread of the disease occur in closed populations (no migration) so that outside influences are ignored.

In modeling the spread of respiratory diseases, population (N) are divided into three classes, individuals susceptible (susceptible), a class of individuals infected (Infected) and individual classes recovered (Recovered). Total population susceptible (S) will increase because of the birth of qN and will be reduced because of the death of μS. Direct contact with an infected individual (I) cause susceptible individuals in the population (S) will become infected and the infection will be entered into the infected population (I). This leads to reduced populations susceptible (S) for the transmission of βSI/N. Class of infection (I) states individuals infected and can transmit the disease to other individuals. This increase in population due to transmission βSI/N and declining population caused by mortality due to other factors and due to respiratory diseases (μ + α) I. And a reduction in the population (I) because of an infected individual can recover ϕI.

Class recover (R) is an individual who recovered from the disease ARI who have temporary immunity. This is because of the increasing population of individuals who recover from infection by φI. And reduced the population to recover (R) is also caused by the death of μR. Due to decreased immunity to diseases ARI, the individuals in the population to recover (R) is assumed to be contracted back into the individual susceptible (S). This can lead to reduced populations to recover (R) of γR. Class (S) is a class (S) for the next year.

Based on the above assumptions, it can set up a transfer diagram SIRS epidemic models (Ma and Li, 2007), as follow:

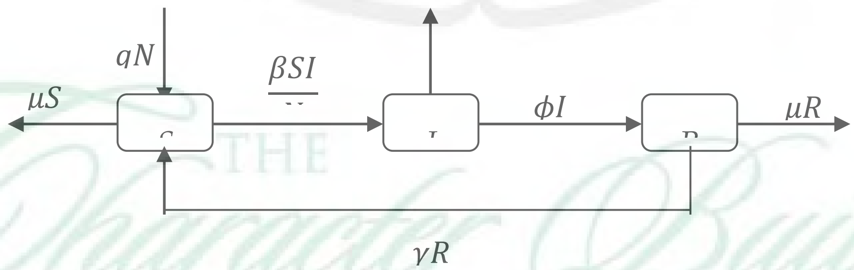

Figure 1. Diagram of SIRS model

Based on the above diagram obtained by the system of ordinary differential equations with three dependent variables were successively claimed the rate of change in the density of susceptible class, the class is infected and cured classes, namely:

$$\frac{dS}{dt} = qN - \frac{\beta SI}{N} - \mu S + \gamma R, \quad (1)$$

$$\frac{dI}{dt} = \frac{\beta SI}{N} - (\mu + \alpha + \phi)I, \quad (2)$$

$$\frac{dR}{dt} = \phi I - (\mu + \gamma)R. \quad (3)$$

The number of individuals in the population expressed N (t) = N, with N = S + I + R, as a result,

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = qN - \mu N - \alpha I. \quad (4)$$

So the number of population will vary depending on the time. System in equation (1) - (3) can be simplified by assuming:

$$x = \frac{S}{N}, y = \frac{I}{N}, z = \frac{R}{N}$$

thus obtained:

$$\frac{dx}{dt} = q - \beta xy - \mu x + \gamma z, \quad (5)$$

$$\frac{dy}{dt} = \beta xy - (\mu + \alpha + \phi)y, \quad (6)$$

$$\frac{dz}{dt} = \phi y - (\mu + \gamma)z. \quad (7)$$

3. Stability Analysis SIRS Model

3.1. Equilibrium point of SIRS Model

Equilibrium point of SIRS model consists of equilibrium point for free-disease E_0 and endemic E_e. Equilibrium point for free-disease means a population in certain area is free from disease. While the equilibrium point for endemic means an outbreak in the population.
To find equilibrium point in the system of equations (5) - (7) is made in a constant position with respect to time conditions,

$$\frac{dx}{dt} = 0, \quad \frac{dy}{dt} = 0, \quad \frac{dz}{dt} = 0.$$

then we have

$$q - \beta xy - \mu x + \gamma z = 0, \quad (8)$$

$$\beta xy - (\mu + \alpha + \phi)y = 0, \quad (9)$$

$$\phi y - (\mu + \gamma)z = 0, \quad (10)$$

Firstly, we will determine the equilibrium point for free-disease by substitute y = 0 into equation (8) - (10) then we obtain x = q/μ and z = 0. So acquired equilibrium point for free-disease is E_0=((q)/μ,0,0). The next we will determine the equilibrium point endemic, because there is an outbreak of disease in the population, it means y ≠ 0.
From equation (9) obtained relationship:
$(\beta x - (\mu + \alpha + \phi))y = 0.$

Because y ≠ 0 then

$$\beta x-(\mu+\alpha+\phi) = 0,$$

$$x = \frac{(\mu+\alpha+\phi)}{\beta}. \quad \textbf{(11)}$$

Furthermore, from the equation (10) was obtained:

$$\phi y-(\mu+\gamma)z = 0,$$

$$z = \frac{\phi y}{(\mu+\gamma)}. \quad \textbf{(12)}$$

with $x=\frac{(\mu+\alpha+\phi)}{\beta}$ and $z=\frac{\phi y}{(\mu+\gamma)}$ it substituted to Equation (3.8) is obtained:

$$q-\beta xy-\ \mu x+\gamma z = 0,$$

$$q-\beta\left(\frac{\mu+\alpha+\phi}{\beta}\right)y-\ \mu\left(\frac{\mu+\alpha+\phi}{\beta}\right)+\gamma\left(\frac{\phi y}{(\mu+\gamma)}\right) = 0,$$

$$q-(\mu+\alpha+\phi)y-\frac{\mu(\mu+\alpha+\phi)}{\beta}+\frac{\gamma\phi y}{(\mu+\gamma)} = 0,$$

$$q-\frac{\mu(\mu+\alpha+\phi)}{\beta}-y\left((\mu+\alpha+\phi)-\frac{\gamma\phi}{(\mu+\gamma)}\right) = 0,$$

$$\frac{q\beta-\mu(\mu+\alpha+\phi)}{\beta}-y\left(\frac{(\mu+\alpha+\phi)(\mu+\gamma)-\gamma\phi}{(\mu+\gamma)}\right) = 0,$$

$$y = \frac{q\beta-\mu(\mu+\alpha+\phi)}{\beta}\left(\frac{(\mu+\gamma)}{(\mu+\alpha+\phi)(\mu+\gamma)-\gamma\phi}\right),$$

$$y = \frac{(\mu+\gamma)[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[(\mu+\alpha+\phi)(\mu+\gamma)-\gamma\phi]},$$

$$y = \frac{(\mu+\gamma)[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[(\mu^2+\mu\gamma+\alpha\mu+\alpha\gamma+\phi\mu+\phi\gamma)-\gamma\phi]},$$

$$y = \frac{(\mu+\gamma)[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}.$$

with $y=\frac{(\mu+\gamma)[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}$ it substituted to the equation (12) was obtained

$$z=\frac{\phi y}{(\mu+\gamma)},$$

$$z=\frac{\phi[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}.$$

Thus obtained equilibrium point for endemic,

$$E_e=(x_e,y_e,z_e) = \left(\frac{(\mu+\alpha+\phi)}{\beta},\frac{(\mu+\gamma)[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]},\frac{\phi[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right).$$

3.2. Stability Analysis at Equilibrium Point of SIRS Model for ARI

To analyze the stability of SIRS models, the first step is linearized model of SIRS. The equation used is the linearized equation (8) - (10).

$$f(x,y,z) = \frac{dx}{dt} = q - \beta xy - \mu x + \gamma z, \quad (13)$$

$$g(x,y,z) = \frac{dy}{dt} = \beta xy - (\mu + \alpha + \phi)y, \quad (14)$$

$$h(x,y,z) = \frac{dz}{dt} = \phi y - (\mu + \gamma)z. \quad (15)$$

Linearization has been done by the Jacobian matrix $J(f(x,y,z))$.

$$J(f(x,y,z)) = \begin{bmatrix} \frac{df(x,y,z)}{dx} & \frac{df(x,y,z)}{dy} & \frac{df(x,y,z)}{dz} \\ \frac{dg(x,y,z)}{dx} & \frac{dg(x,y,z)}{dy} & \frac{dg(x,y,z)}{dz} \\ \frac{dh(x,y,z)}{dx} & \frac{dh(x,y,z)}{dy} & \frac{dh(x,y,z)}{dz} \end{bmatrix} \quad \textbf{(16)}$$

Jacobian matrix of the system in equation (13) - (15) is:

$$J(f(x,y,z)) = \begin{bmatrix} -\beta y - \mu & -\beta x & \gamma \\ \beta y & \beta x - (\mu + \alpha + \phi) & 0 \\ 0 & \phi & -(\mu + \gamma) \end{bmatrix} \quad \textbf{(17)}$$

3.2.1. Stability Analysis of SIRS model for Free Disease

Given equilibrium point for free-disease E_0=((q)/μ,0,0) evaluated the Jacobian matrix (17) thus obtained:

$$J(E_0) = \begin{bmatrix} -\mu & -\beta\left(\frac{q}{\mu}\right) & \gamma \\ 0 & \beta\left(\frac{q}{\mu}\right) - (\mu + \alpha + \phi) & 0 \\ 0 & \phi & -(\mu + \gamma) \end{bmatrix}$$

Eigenvalues of Jacobian matrix is

$$J(E_0) = \begin{bmatrix} -\mu - \lambda & -\beta\left(\frac{q}{\mu}\right) & \gamma \\ 0 & \beta\left(\frac{q}{\mu}\right) - (\mu + \alpha + \phi) - \lambda & 0 \\ 0 & \phi & -(\mu + \gamma) - \lambda \end{bmatrix}$$

$$= (-\mu - \lambda)\left(\beta\left(\frac{q}{\mu}\right) - (\mu + \alpha + \phi) - \lambda\right)(-(\mu + \gamma) - \lambda)$$

$$= -\lambda^3 + \left(-\gamma - 3\mu - \phi - \alpha + \frac{\beta q}{\mu}\right)\lambda^2 + \left(-\gamma\phi - 2\gamma\mu + \frac{\gamma\beta q}{\mu} - 2\mu\alpha - 3\mu^2 - 2\mu\phi - \gamma\alpha + 2q\beta\right)\lambda - \mu^2\gamma - \mu\gamma\phi - \mu\gamma\alpha - \mu^3 - \mu^2\phi - \mu^2\alpha + \gamma q\beta + \mu q\beta$$

Eigenvalues equation can be written as follows:

-λ^3+(-γ-3μ-ϕ-α+βq/μ) λ^2+ (-γϕ-2γμ+γβq/μ-2μα-3μ^2-2μϕ-γα+2qβ)λ-μ^2 γ-μγϕ-μγα-μ^3-μ^2 ϕ〖-μ〗^2 α+γqβ+μqβ=0.

According to Diekmann et al. (1990) to get the basic reproduction number (R_0), by taking the constants of the equation are the eigenvalues.

$$\begin{aligned}
-\mu^2\gamma - \mu\gamma\phi - \mu\gamma\alpha - \mu^3 - \mu^2\phi - \mu^2\alpha + \gamma q\beta + \mu q\beta &= 0, \\
\mu^2\gamma + \mu\gamma\phi + \mu\gamma\alpha + \mu^3 + \mu^2\phi + \mu^2\alpha &= \gamma q\beta + \mu q\beta, \\
\frac{\gamma q\beta + \mu q\beta}{\mu^2\gamma + \mu\gamma\phi + \mu\gamma\alpha + \mu^3 + \mu^2\phi + \mu^2\alpha} &= 1, \\
\frac{q\beta}{(\mu+\alpha+\phi)\mu} &= 1 = R_0. \qquad \textbf{(18)}
\end{aligned}$$

And the eigenvalues of the Jacobian: $\lambda_1 = -\mu$, $\lambda_2 = -(\mu+\gamma)$, and $\lambda_3 = \beta\left(\frac{q}{\mu}\right) - (\mu+\alpha+\phi)$. E_0 will be asymptotically stable if $\lambda_n < 0$ for $n = 1,2,3$. For $\mu > 0$ resulting in value $\left.\begin{matrix}\lambda_1 = -\mu \\ \lambda_2 = -(\mu+\gamma)\end{matrix}\right\} < 0$. If $\lambda_3 < 0$ then $\beta\left(\frac{q}{\mu}\right) < (\mu+\alpha+\phi)$. $\beta\left(\frac{q}{\mu}\right) < (\mu+\alpha+\phi)$ we have,

$$\begin{aligned}
\frac{\beta\left(\frac{q}{\mu}\right)}{(\mu+\alpha+\phi)} &< 1, \\
\frac{q\beta}{(\mu+\alpha+\phi)\mu} &= R_0 < 1. \qquad \textbf{(19)}
\end{aligned}$$

Therefore, point E_0 will be asymptotically stable for $R_0 \leq 1$. However, if $R_0 \geq 1$ then the point will be unstable.

3.1.1. Analysis Stability of SIRS Model for Endemic

Equilibrium point for endemic $E_e = \left(\frac{(\mu+\alpha+\phi)}{\beta}, \frac{(\mu+\gamma)[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}, \frac{\phi[q\beta-\mu(\mu+\alpha+\phi)]}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right)$ evaluated on Jacobian matrix **(17)** thus we have:

$$J(f(E_e)) = \begin{bmatrix} -\beta y - \mu & -\beta x & \gamma \\ \beta y & \beta y - (\mu+\alpha+\phi) & 0 \\ 0 & \phi & -(\mu+\gamma) \end{bmatrix}$$

Equilibrium point E_e can also be determined using the basic definition of the basic reproduction number (R_0) into:

$$E_e = \left(\frac{(\mu+\alpha+\phi)}{\beta}, \frac{(\mu+\alpha+\phi)(\mu+\gamma)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}, \frac{\phi(\mu+\alpha+\phi)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right)$$

$$J(f(E_e)) = \begin{bmatrix} -\beta\left(\frac{(\mu+\alpha+\phi)(\mu+\gamma)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right) - \mu & -\beta\left(\frac{(\mu+\alpha+\phi)}{\beta}\right) & \gamma \\ \beta\left(\frac{(\mu+\alpha+\phi)(\mu+\gamma)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right) & \beta\left(\frac{(\mu+\alpha+\phi)}{\beta}\right) - (\mu+\alpha+\phi) & 0 \\ 0 & \phi & -(\mu+\gamma) \end{bmatrix}$$

$$= \begin{bmatrix} -\left(\frac{(\mu+\alpha+\phi)(\mu+\gamma)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right)-\ \mu & (\mu+\alpha+\phi) & \gamma \\ \left(\frac{(\mu+\alpha+\phi)(\mu+\gamma)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right) & 0 & 0 \\ 0 & \phi & -(\mu+\gamma) \end{bmatrix}$$

Eigenvalues of Jacobian matrix is

$$= \begin{bmatrix} -\left(\frac{(\mu+\alpha+\phi)(\mu+\gamma)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right)-\ \mu-\lambda & (\mu+\alpha+\phi) & \gamma \\ \left(\frac{(\mu+\alpha+\phi)(\mu+\gamma)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right) & -\lambda & 0 \\ 0 & \phi & -(\mu+\gamma)-\lambda \end{bmatrix}$$

$$= \left(-\left(\frac{(\mu+\alpha+\phi)(\mu+\gamma)(R_0-1)}{\beta[\phi\mu+(\mu+\alpha)(\mu+\gamma)]}\right)-\ \mu-\lambda\right)(-\lambda)(-\mu-\gamma-\lambda).$$

From equation (20), endemic point will only appear if $R_0 \geq 1$ while eigenvalues λ_1, λ_2 and λ_3 will be real numbers or complex numbers with real numbers negative if $R_0 \geq 1$.

3.3 BASIC REPRODUCTION NUMBERS

In epidemiology, the rate of spread of an infectious disease can be measured by a value called the Basic Reproductive Numbers (R_0). It is said that an area free from infectious respiratory disease, the value of R_0 is R_0≤1. In this case each patient is only able to spread the disease to less than or equal to one new patient, so that eventually the disease will disappear. Whereas, if R_0≥1 so every patient can spread the disease to an average of more than or equal to one new patients, so in the end there will be endemic. Basic reproduction number (R_0) can be obtained from equation (16) and rewritten as follows:

$$R_0 = \frac{q\beta}{(\mu+\alpha+\phi)\mu}$$

R_0≤1 occurred if qβ≤(μ+α+ϕ)μ while R_0≥1 if qβ≥(μ+α+ϕ)μ. Based on the results which have obtained R_0, to make R_0≤1, the denominator must be greater than the numerator. Deaths due to other factors and mortality due to respiratory infection cannot be upgraded. Therefore, you need to do is cure or treatment for patients with acute respiratory infection that healing rate (φ) will be increased. In addition the rate of transmission (β) respiratory disease should also be lowered, thus the incidence of respiratory disease will be reduced so that the disease can be controlled from the state of the epidemic. So it can be said of this analysis will be known most significant influencing parameters or all the parameters in the model of the spread of respiratory diseases is the parameter β and ϕ.

4. SIMULATION

In this section, Data of ARI have taken from the Maraja Bah Jambi Java District in 2013-2014

Table 1. Data of ARI disease from 2013 to 2014 in the Maraja Bah Jambi Java Distirct

Year	***Susceptible* (S)**	***Infected* (I)**	***Recovered* (R)**	**Populasi (N)**
2013	18.911	865	810	20.586
2014	18.855	918	936	20.709

Based on data taken from health centers Maraja Java, birth and death rate of the population is calculated based on population in 2013 and 2014, ie:

- Birth rate (q) = death rate (μ)

1. Relevant Finding

$$\mu = q \quad = \frac{\text{the number of births}}{\text{total population}},$$

$$= \frac{\text{a population of 2014} - \text{population of 2013}}{\text{total population}},$$

$$= \frac{20.709 - 20.586}{20.586},$$

$$= \frac{123}{20.586},$$

$$= 0{,}006.$$

- The rate of disease transmission (β)

$$\beta \quad = \frac{number\ of\ infected\ in\ 2013}{number\ of\ susceptible\ in\ 2013},$$

$$= \frac{865}{18.911},$$

$$= 0{,}046.$$

According to Ma and Li (2007), in general, if T is the time spent in class, then the rate of individuals who leave the class is 1/T. Individuals infected can transmit the disease to other individuals. Average period of infectivity for respiratory diseases is 14 days. After 14 days it will leave the class of infected individuals to recovered, so the pace of the individual is

$$\phi = \frac{1}{\text{infectivity period}} = \frac{1}{14 \text{ days}} = 0.071.$$

After passing through the infectivity and into the recovered grade, an individual has a period of temporary immunity to diseases ARI is for 3 months or 90 days and will leave the class recovered and returned to the individual vulnerable to diseases ARI with individual rate is

$$\gamma = \frac{1}{immune\ period} = \frac{1}{90 \text{ days}} = 0.011.$$

Based on data taken from health centers Maraja Java in 2013 and 2014, there is no individual who died of respiratory disease, the magnitude of the rate of death due to ARI disease is (α)=0. Based on the values of parameters and data, model simulations computed using Maple software. The obtained formulation SIRS epidemic models on the spread of respiratory diseases in the Maraja Bah Jambi Java District as follows:

$$\frac{dx}{dt} = 0.006 - 0.046xy - 0.006x + 0.011z, \qquad \textbf{(20)}$$

$$\frac{dy}{dt} = 0.046xy - 0{,}077y, \qquad \textbf{(21)}$$

$$\frac{dz}{dt} = 0{,}071y - 0.017z. \qquad \textbf{(22)}$$

with initial condition.

$x_0 = \frac{18.911}{20.586} = 0.9186, y_0 == \frac{865}{20.586} = 0.0420$, and $z_0 = \frac{810}{20.586} = 0{,}0393$

Table 2. Simulation result of ARI diseasesince 2013 to 2014

Year	***Susceptible*** **(S)**	***Infected*** **(I)**	***Recovered*** **(R)**
2013	18.911	865	810
2014	18.894	835	856

Infected population of ARI disease in Maraja Bah Jambi java district plotted in below figure

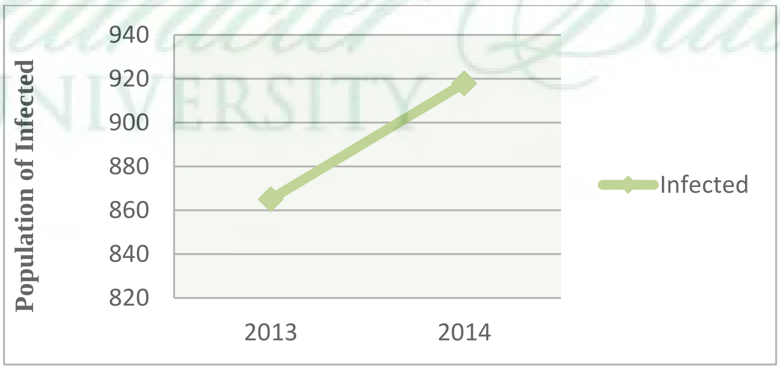

Figure 1. Infected population in Jawa Maraja Bah Jambi district

Approximate solution for Susceptible, Infected, dan Recovered which is shown in figures (2) – (4).

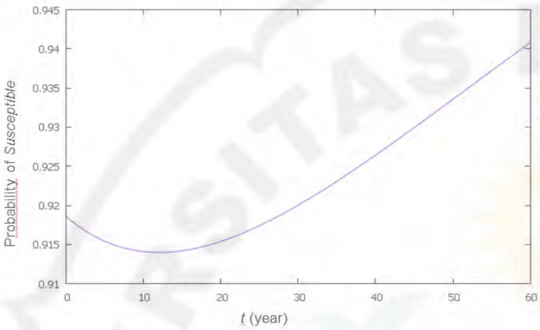

Figure 2. Approximate solution of probability of susceptible

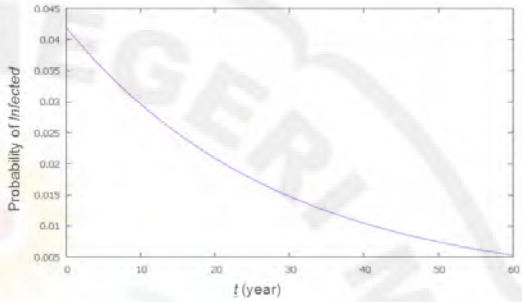

Figure 3. Approximate solution of probability of Infected

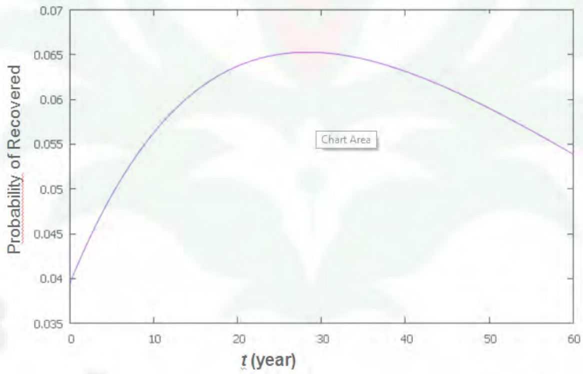

Figure 4. Approximate solution of probability of Recovered

Figure 2 is generated from numerical simulation with Maple for SIRS models compared with the real data (Figure 1) of the Health Center Maraja Bah Jambi Java can be described as follow:

a. Referring to the real data in the Maraja Bah Jambi Java District, the number of cases of ARI in Maraja Bah Jambi Java District is increased by 53 deaths from 2013-2014, while the results of the simulation model of SIRS in Figure 3 shows the number of ARI cases in the district is declined from the year 2013-2014 amount 30 individuals and it will be continuously declined until 60 years later.

b. Meanwhile, the number of susceptible populations from the simulation, in Figure 2 shows that the susceptible population is declined in the 10th until the 12th year and continued to rise until the 60th year. This is because individuals who have recovered have temporary immunity from the ARI and will again become susceptible individuals. The susceptible individuals are infected and entered in the population infected, so the infected population has increased at the beginning (Figure 3). And over time the number of infected populations will be declined. This is because individuals infected with respiratory diseases have been cured and recovered into the population, so that the recovered grade has increased and the passage of time will decrease as the individual recovered at the end of which has a temporary immunity will go back to being susceptible individuals (Figure 4). However, at certain times the number of individuals susceptible, infected, or recovered unchanged. This state is called the equilibrium state. In the equilibrium state of disease will always be there indefinitely.

At equilibrium conditions, the disease will not disappear for a time $t \to \infty$. The next, it will be determined the stability of the equilibrium point for free-disease E_0. The basic reproduction number $R_0 = 0.59$. Therefore, the disease-free equilibrium point is asymptotically because R_0 values satisfy the equation (17). $R_0 \leq 1$ value which means the disease will disappear in the population with the number of patients within normal limits. Basic reproduction number can be used to determine whether the disease is endemic or not. The disease will remain there until the time of infinite (endemic) if $R_0 \geq 1$. Based on the above results indicated that the Maraja Bah Jambi Java District is not endemic, the spread of respiratory diseases will not spread more widely and will disappear within a certain time limit.

5. CONCLUSION

SIRS models used in the spread of respiratory diseases in the Maraja Bah Jambi Java District. Analysis and simulation models of SIRS above were obtained the following results:

a. The SIRS models have two equilibrium points that is free-disease equilibrium point $E_0 = (1,0,0)$ which is asymptotically stable and equilibrium point for endemic $E_e = (1.673913,-0.130187,-0.543725)$ is unstable.

b. The value of the basic reproduction number $R_0 = 0.59 \leq 1$ which means that every individual infected, only spread to one other individual. It means that the area the Maraja Bah Jambi Java District is not endemic or is not harmful.

c. Results of numerical simulation for SIRS model using fourth order Runge-Kutta method for ARI cases is declined in 2013-2014.

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