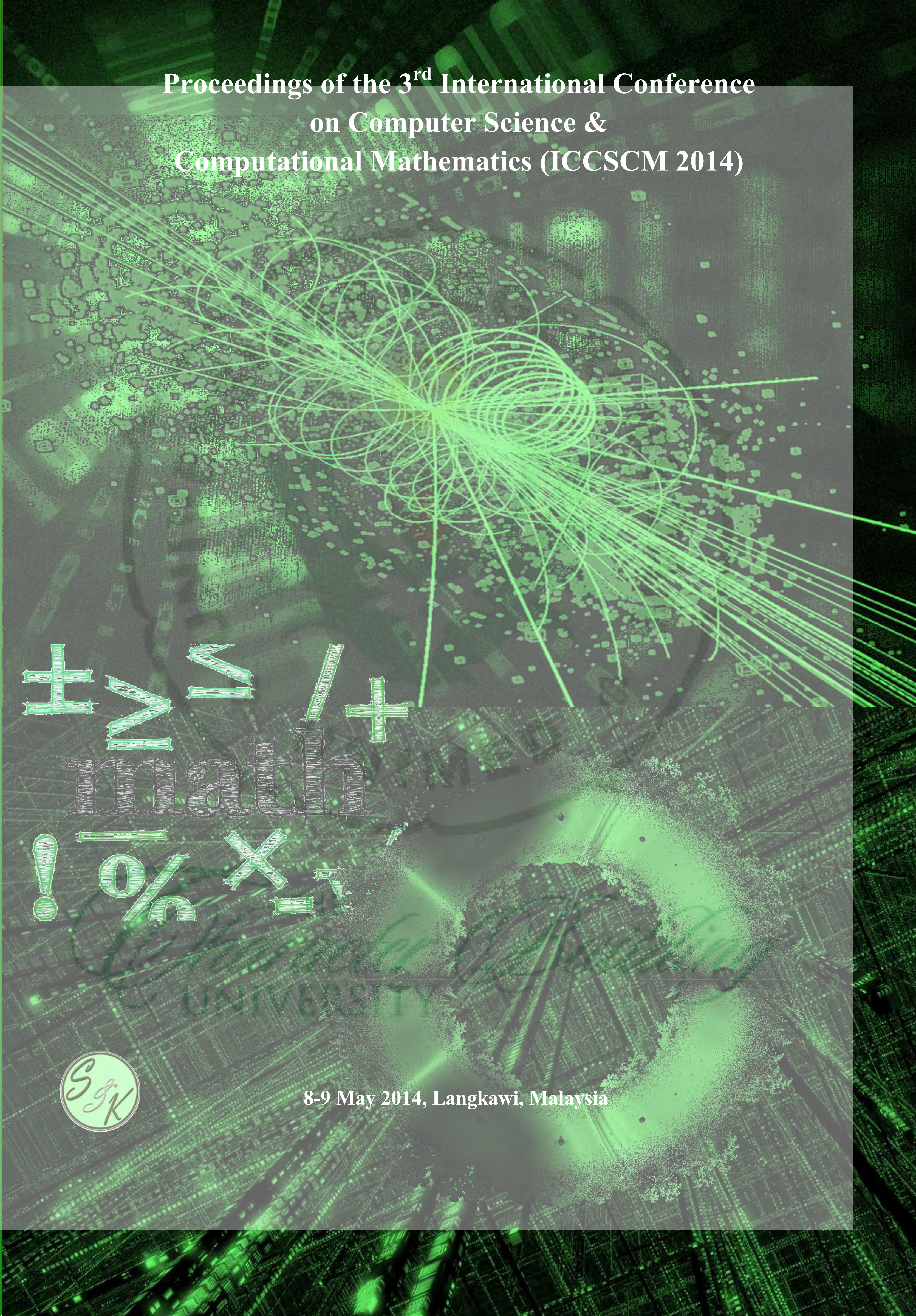
Proceedings of the 3rd International Conference on Computer Science & Computational Mathematics (ICCSCM 2014)



8-9 May 2014, Langkawi, Malaysia

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Preface

ICCSCM 2014 (The 3rd International Conference on Computer Science & Computational Mathematics) has aimed to provide a platform to discuss computer science and mathematics related issues including Algebraic Geometry, Algebraic Topology, Approximation Theory, Calculus of Variations, Category Theory; Homological Algebra, Coding Theory, Combinatorics, Control Theory, Cryptology, Geometry, Difference and Functional Equations, Discrete Mathematics, Dynamical Systems and Ergodic Theory, Field Theory and Polynomials, Fluid Mechanics and Solid Mechanics, Fourier Analysis, Functional Analysis, Functions of a Complex Variable, Fuzzy Mathematics, Game Theory, General Algebraic Systems, Graph Theory, Group Theory and Generalizations, Image Processing, Signal Processing and Tomography, Information Fusion, Integral Equations, Lattices, Algebraic Structures, Linear and Multilinear Algebra; Matrix Theory, Mathematical Biology and Other Natural Sciences, Mathematical Economics and Financial Mathematics, Mathematical Physics, Measure Theory and Integration, Neutrosophic Mathematics, Number Theory, Numerical Analysis, Operations Research, Optimization, Operator Theory, Ordinary and Partial Differential Equations, Potential Theory, Real Functions, Rings and Algebras, Statistical Mechanics, Structure Of Matter, Topological Groups, Wavelets and Wavelet Transforms, 3G/4G Network Evolutions, Ad-Hoc, Mobile, Wireless Networks and Mobile Computing, Agent Computing & Multi-Agents Systems, All topics related Image/Signal Processing, Any topics related Computer Networks, Any topics related ISO SC-27 and SC-17 standards, Any topics related PKI(Public Key Intrastructures), Artifial Intelligences(A.I.) & Pattern/Image Recognitions, Authentication/Authorization Issues, Biometric authentication and algorithms, CDMA/GSM Communication Protocols, Combinatorics, Graph Theory, and Analysis of Algorithms, Cryptography and Foundation of Computer Security, Data Base(D.B.) Management & Information Retrievals, Data Mining, Web Image Mining, & Applications, Defining Spectrum Rights and Open Spectrum Solutions, E-Comerce, Ubiquitous, RFID, Applications, Fingerprint /Hand/Biometrics Recognitions and Technologies, Foundations of High-performance Computing, IC-card Security, OTP, and Key Management Issues, IDS/Firewall, Anti-Spam mail, Anti-virus issues, Mobile Computing for E-Commerce, Network Security Applications, Neural Networks and Biomedical Simulations, Quality of Services and Communication Protocols, Quantum Computing, Coding, and Error Controls, Satellite and Optical Communication Systems, Theory of Parallel Processing and Distributed Computing, Virtual Visions, 3-D Object Retrievals, & Virtual Simulations, Wireless Access Security, etc.

The success of ICCSCM 2014 is reflected in the received papers from authors around the world from several countries which allows a highly multinational and multicultural idea and experience exchange.

The accepted papers of ICCSCM 2014 are published in this Book. Please check www. iccscm.com for further news. They will also be included in the electronic library of the www.sandkrs.com.

A conference such as ICCSCM 2014 can only become successful using a team effort, so herewith we want to thank the International Technical Committee and the Reviewers for their efforts in the review process as well as their valuable advices. We are thankful to all those who contributed to the success of ICCSCM 2014.

The Secretary

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Variational iteration and homotopy perturbation methods for obtaining an approximate solution of SEIR model of dengue fever in South Sulawesi

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***Abstract*:** In this paper, the susceptible-exposed–infected–recovered (SEIR) model of dengue fever disease in South Sulawesi is discussed. The SIR model is formed by a system of nonlinear differential equation. We shall comparevariational iteration method (VIM) againsthomotopy perturbation method (HPM). The Lagrange multiplier is investigated for VIM and the He's polynomial approach for HPM is used. The two methods are the alternative methods to obtainthe approximate solutions of the SEIR model. Additional comparison will be made against the conventional numerical method, fourth Runge-Kutta method (RK4). From the result, VIM solution is more accurate than HPM solution for long time interval when it compared to fourth order Runge–Kutta (RK4) and plotting of real data.

***Keywords*:** Variational iteration method, Homotopy perturbation method, Lagrange multiplier, He polynomial, SEIR Model.

1. Introduction

Variational iteration method (VIM) proposed by He [1]. The essential idea of the method is to investigate the Lagrange multiplier for correction functional in the VIM. This technique has been employed to solve a large variety of linear and nonlinear problem.Yulita and collegues [2-4] obtained the approximate solution of fractional heat and wave-like equations, fractional Zakharov-Kuznetsov equation and Fractional Rosenao-Hayman equation using VIM. Yulita [5] modified the VIM to find the approximate solution of fractional Biochemical Reaction model. Rafei et al. [6] applied VIM for solving the epidemic model and the prey and predator problem.

Another approximate analytical method was introduced by He [7,8] such as homotopy perturbation method (HPM). The basic idea of HPM is to introduce a homotopy parameter p which takes value from 0 to 1. when the perturbation parameter $p = 0$, the system reduce toa linear system of equations, which normally admits to rather simple solution.Whereas,$p = 1$, the system takes the original form of the equation and final stage of deformation gives the desired solution. One of the most remarkable features of the HPM is that usually just a few perturbation terms are sufficient for obtaining a reasonablyaccurate solution.Khan et al. [9] applied HPM to Vector Host Epidemic Model with Non-Linear Incidences andGhotbi et al [10] used the HPM and VIM to SIR epidemic model. Recently, Islam et al [11] obtained the analytical solution of an SEIV epidemic model by HPM. The procedure of the two methods for the SIR model will be discussed later. In this paper, the VIM and HPM solutions also matched with the empirical data in [12] to show the accuracy of the methods.

Dengue fever is regarded as a serious infectious disease threatening about 2.5 billion people all over the world, especially in tropical countries. Dengue fever has become a major epidemic disease in Southeast Asia. Such an epidemic arises from climate change and is made worse by the population's lack of knowledge about and awareness of dengue fever, so that dengue fever may become endemic [12]. Thus, building model for the dengue fever is important.
Mathematical models for dengue fever have investigated compartment dynamics using Susceptible, Infected, and Removed (SIR) models [13]-[18]; these models have only scrutinized the formulation of the model. Side and Noorani [12] have modified the models in [12] and [119] and applied the collected real data reported by the Ministry of Health in South Sulawesi, Indonesia (KKRI) [20]. Side and Noorani [12] also was match the empirical data with the model simulation. Hence, the SIR model presented in [12] is intended to be a trusted reference and as a control tool in dealing with dengue fever in South Sulawesi. To find the spreading number of populations in this model [12] using semi-numerical method is interested to investigate. The precise method must be chosen to solve this model.
Side and Noorani [12] defined a SIR model of dengue fever in the following equation

$$\frac{dx}{dt} = \mu_h(1-x) - p_1 x + \alpha x z, \quad (1)$$

$$\frac{dy}{dt} = (\alpha u + p_1)z - (\mu_h + \varphi_h)y, \quad (2)$$

$$\frac{dz}{dt} = \varphi_h y - (\mu_h + \gamma_h + \alpha_h)z, \quad (3)$$

$$\frac{du}{dt} = \gamma_v(1-v-u)z - (\mu_v + \delta_v)u \quad (4)$$

$$\frac{dv}{dt} = \delta_v u - \mu_v v \quad (5)$$

where$x = \frac{S_h}{N_h}$, $y = \frac{I_h}{N_h}$, $z = \frac{I_h}{N_h}$, $u = \frac{E_v}{N_v}$, $v = \frac{I_v}{N_v} = \frac{I_v}{A/\mu_v}$, with $0 \le x, y, z, u, v \le 1$ and $\alpha = \frac{b\beta_h A}{\mu_v N_h}$, $\mu_h = 0.000046$, $p_1 = 0.09$, $\varphi_h = 0.1667$, $\gamma_h = 0.3288330$, $\alpha = 0.0000002$, $\delta_v = 0.1428000$. $\mu_v = 0.0323000$, $b\beta_h = 0.75$ and $b\beta_v = 0.375$. According to Side and Noorani [12], N_h is the human population, S_h is people who may potentially get infected with dengue virus, $I_h$is people who are infected with dengue. $R_h$is people who have recovered, and E_h indicates people who exposed of virus infection.The vector population of mosquitoes (N_v) is divided into two groups: mosquitoes that may potentially become infected with dengue virus (susceptible; S_v) and mosquitoes that are infected with dengue virus (I_v). $b\beta_h$is sufficient rate of correlation of vector population to human population.

2. Homotopy Perturbation Method

To implement HPM, firstly, we write a general system of differential equationin the operator form:

$$\frac{du_1}{dt} + g_1(t, u_1, u_2, \cdots, u_m) = f_1(t), \quad (6)$$

$$\frac{du_2}{dt} + g_2(t, u_1, u_2, \cdots, u_m) = f_2(t), \quad (7)$$

$$\vdots$$

$$\frac{du_m}{dt} + g_m(t, u_1, u_2, \cdots, u_m) = f_m, \quad (8)$$

subject to the initial conditions

$$u_1(t_0) = c_1, \qquad u_2(t_0) = c_2, \quad \cdots u_m(t_0) = c_m. \quad (9)$$

Then we write system (6)–(8) in the following operator form:

$$L(u_1) + N_1(u_1, u_2, \cdots, u_m) - f_1(t) = 0, \quad (10)$$

$$L(u_2) + N_2(u_1, u_2, \cdots, u_m) - f_2(t) = 0, \quad (11)$$

$$\vdots$$

$$L(u_2) + N_2(u_1, u_2, \cdots, u_m) - f_2(t) = 0, \quad (12)$$

subject to the initial conditions (9), where $L = d/dt$is linear operator and $N_1, N_2, \ldots, N_m$are nonlinear operators. We shall next present the solution approaches of (10)–(12) based on the standard HPM.

According to HPM, we construct a homotopy for (10)–(12) which satisfies the following relations:

$$L(u_1) - L(v_1) + pL(v_1) + p[N_1(u_1, u_2, \cdots, u_m) - f_1(t)] = 0, \quad (13)$$

$$L(u_2) - L(v_2) + pL(v_2) + p[N_2(u_1, u_2, \cdots, u_m) - f_2(t)] = 0, \quad (14)$$

$$\vdots$$

$$L(u_m) - L(v_m) + pL(v_m) + p[N_m(u_1, u_2, \cdots, u_m) - f_m(t)] = 0, \quad (15)$$

where$p \in [0,1]$is an embedding parameter and $v_1, v_2, \ldots, v_m$are initial approximations which satisfying the given conditions. It is obvious that when the perturbation parameter $p = 0$, Eqs. (11)–(13) become a linear system of equations and when $p = 1$ we get the original nonlinear system of equations. Let us take the initial approximations as follows:

$$u_{1,0}(t) = v_1(t) = u_1(t_0) = c_1, \quad (16)$$

$$u_{2,0}(t) = v_2(t) = u_2(t_0) = c_2, \quad (17)$$

$$\vdots$$

$$u_{m,0}(t) = v_m(t) = u_m(t_0) = c_m. \quad (18)$$

And

$$u_1(t) = u_{1,0}(t) + pu_{1,1}(t) + p^2 u_{1,2}(t) + \cdots, \quad (19)$$

$$u_2(t) = u_{2,0}(t) + pu_{2,1}(t) + p^2 u_{2,2}(t) + \cdots, \quad (20)$$

$$\vdots$$

$$u_m(t) = u_{m,0}(t) + pu_{m,1}(t) + p^2 u_{m,2}(t) + \cdots, \quad (21)$$

where$u_{i,j}$ $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots)$ are functions yet to be determined. Substituting (14)–(19) into (11)–(13) and arranging the coefficients of the same powers of p, we get

$$L(u_{1,1}) + L(v_1) + N_1(u_{1,0}, u_{2,0}, \cdots, u_{m,0}) - f_1 = 0, \ u_{1,1}(t_0) = 0, \quad (22)$$

$$L(u_{2,1}) + L(v_2) + N_2(u_{1,0}, u_{2,0}, \cdots, u_{m,0}) - f_2 = 0, u_{2,1}(t_0) = 0, \quad (23)$$

$$\vdots$$

$$L(u_{m,1}) + L(v_m) + N_m(u_{1,0}, u_{2,0}, \cdots, u_{m,0}) - f_m = 0, u_{m,1}(t_0) = 0, \quad (24)$$

and

$$L(u_{1,2}) + N_1(u_{1,0}, u_{2,0}, \cdots, u_{m,0}) - f_1 = 0, \ u_{1,2}(t_0) = 0, \quad (25)$$

$$L(u_{2,2}) + N_2(u_{1,0}, u_{2,0}, \cdots, u_{m,0}) - f_2 = 0, u_{2,2}(t_0) = 0, \quad (26)$$

$$\vdots$$

$$L(u_{m,2}) + N_m(u_{1,0}, u_{2,0}, \cdots, u_{m,0}) - f_m = 0, u_{m,2}(t_0) = 0, \quad (27)$$

etc. We solve the above systems of equations for the unknowns $u_{i,j}$ $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots)$ by applying the inverse operator

$$L^{-1}(\cdot) = \int_0^t (\cdot)\, dt. \quad (28)$$

Therefore, according to HPM the n-term approximations to the solutions of (8)–(10) can be expressed as

$$\phi_{1,n}(t) = u_1(t) = \lim_{p \to 1} u_1(t) = \sum_{k=0}^{n-1} u_{1,k}(t), \quad (29)$$

$$\phi_{2,n}(t) = u_2(t) = \lim_{p \to 1} u_2(t) = \sum_{k=0}^{n-1} u_{2,k}(t), \quad (30)$$

$$\vdots$$

$$\phi_{m,n}(t) = u_m(t) = \lim_{p \to 1} u_m(t) = \sum_{k=0}^{n-1} u_{m,k}(t), \quad (31)$$

3. Variational Iteration Method (VIM)

To introduce the basic concepts of VIM, we consider the following nonlinear differential equation:

$$Lu_i(t) + Nu_i(t) = g_i(t), \qquad i = 1,2,\cdots,n \qquad (32)$$

where L is a linear operator, N is a nonlinear operator, and $g_i(t)$ is an inhomogeneous term. According the VIM, one can construct a correction functional as follows:

$$u_{i,n+1} = u_{i,n} + \int_0^t \lambda_i(s)\left[Lu_{i,n}(s) + N\widetilde{u_{i,n}}(s) - g_i(t)\right] d\,s, \qquad (33)$$

where $\lambda_i,\ \ i = 1,2,\cdots,n$ are the Lagrange multiplier [21] which can be identified optimally via the variational theory, and $\widetilde{u_{i,n}}(s)$ are considered as restricted variations, i.e. $\delta\widetilde{u_{i,n}}(s) = 0$. Once we have determined the Lagrange multiplier, we use VIM to perform the iteration using the initial approximation, which we choose by linearized solution of the equation which satisfies the initial condition. Therefore, we can successively approximate or even reach the exact solution by using

$$u(t) = \lim_{n\to\infty} u_{i,n}(t) \qquad (34)$$

4. Implementation of VIM

First, we consider the SEIR model which was written in (2)-(5). To apply VIM to SEIR model, we construct the correction functional as follows:

$$\boldsymbol{x_{n+1}(t) = x_n + \int_0^t \lambda_1(s)\left[\frac{dx_n}{dt} - \mu_h(1 - x_n) + p_1x_n - \alpha\tilde{x}_n\tilde{z}_n\right] ds,} \qquad (35)$$

$$\boldsymbol{y_{n+1}(t) = y_n + \int_0^t \lambda_2(s)\left[\frac{dy_n}{dt} - (\alpha\tilde{u}_n + p_1)\tilde{z}_n + (\mu_h + \varphi_h)y_n\right] ds,} \qquad (36)$$

$$\boldsymbol{z_{n+1}(t) = z_n + \int_0^t \lambda_3(s)\left[\frac{dz_n}{ds} - \varphi_h\tilde{y}_n - (\mu_h + \gamma_h + \alpha_h)z_n\right] ds,} \qquad (37)$$

$$\boldsymbol{u_{n+1}(t) = u_n + \int_0^t \lambda_4(s)\left[\frac{du_n}{dt} - \gamma_v(1 - \tilde{v}_n - \tilde{u}_n)\tilde{z}_n + (\mu_v + \delta_v)u_n\right] ds} \qquad (39)$$

$$\boldsymbol{v_{n+1}(t) = v_n + \int_0^t \lambda_5(s)\left[\frac{dv_n}{dt} - \delta_v u_n + \mu_v v_n\right] ds} \qquad (40)$$

where $\lambda_i,\ i = 1,2,3,4,5$ are a general Lagrange multiplier which can be identified optimally via the variational theory and the subscript n indicates the nth,. To obtain the optimal $\lambda(s)$, we proceed as follows:

$$\boldsymbol{\delta x_{n+1} = \delta x_n + \int_0^t \delta\lambda_1(s)\left[\frac{dx_n}{dt} - \mu_h(1 - x_n) + p_1x_n - \alpha\tilde{x}_n\tilde{z}_n\right] ds,} \qquad \textbf{(41)}$$

$$\boldsymbol{\delta y_{n+1} = \delta y_n + \int_0^t \delta\lambda_2(s)\left[\frac{dy_n}{dt} - (\alpha\tilde{u}_n + p_1)\tilde{z}_n + (\mu_h + \varphi_h)y_n\right] ds,} \qquad (42)$$

$$\boldsymbol{\delta z_{n+1} = \delta z_n + \int_0^t \delta\lambda_3(s)\left[\frac{dz_n}{ds} - \varphi_h\tilde{y}_n - (\mu_h + \gamma_h + \alpha_h)z_n\right] ds,} \qquad (43)$$

$$\boldsymbol{\delta u_{n+1}(t) = \delta u_n + \int_0^t \delta\lambda_4(s)\left[\frac{du_n}{dt} - \gamma_v(1 - \tilde{v}_n - \tilde{u}_n)\tilde{z}_n + (\mu_v + \delta_v)u_n\right] ds} \qquad (44)$$

$$\boldsymbol{\delta v_{n+1}(t) = \delta v_n + \int_0^t \delta\lambda_5(s)\left[\frac{dv_n}{dt} - \delta_v u_n + \mu_v v_n\right] ds} \qquad (45)$$

where $\tilde{x}_n, \tilde{y}_n$ and $\tilde{z}_n$ are considered as restricted variations, i.e., $\tilde{x}_n, \tilde{y}_n = 0$ and $\tilde{z}_n = 0$. Then, we have

$$\boldsymbol{\delta x_{n+1} = \delta x_n + \int_0^t \delta\lambda_1(s)\left[\frac{dx_n}{ds} - \mu_h(1 - x_n) + p_1x_n\right] ds,} \qquad (46)$$

$$\boldsymbol{\delta y_{n+1} = y_n + \int_0^t \delta\lambda_2(s)\left[\frac{dy_n}{ds} + (\mu_h + \varphi_h)y_n\right] ds,} \qquad (47)$$

$$\boldsymbol{\delta z_{n+1} = \delta z_n + \int_0^t \delta\lambda_3(s)\left[\frac{dz_n}{ds} - (\mu_h + \gamma_h + \alpha_h)z_n\right] ds,} \qquad (48)$$

$$\boldsymbol{\delta u_{n+1}(t) = \delta u_n + \int_0^t \delta\lambda_4(s)\left[\frac{du_n}{dt} + (\mu_v + \delta_v)u_n\right] ds} \qquad (49)$$

$$\boldsymbol{\delta v_{n+1}(t) = \delta v_n + \int_0^t \delta\lambda_5(s)\left[\frac{dv_n}{dt} + \mu_v v_n\right] ds} \qquad (50)$$

or

$$\boldsymbol{\delta x_{n+1} = \delta x_n + \int_0^t \delta\lambda_1(s)\frac{dx_n}{ds} + \delta\lambda_1(s)(\mu_h + p_1)x_n ds,} \qquad \textbf{(51)}$$

$$\delta y_{n+1} = y_n + \int_0^t \delta\lambda_2(s)\frac{dy_n}{ds} + \delta\lambda_2(s)(\mu_h+\varphi_h)y_n\ ds, \quad (52)$$

$$\delta z_{n+1} = \delta z_n + \int_0^t \delta\lambda_3(s)\frac{dz_n}{ds} - \delta\lambda_3(s)(\mu_h+\gamma_h+\alpha_h)z_n\ ds, \quad (53)$$

$$\delta u_{n+1}(t) = \delta u_n + \int_0^t \delta\lambda_4(s)\frac{du_n}{dt} + \delta\lambda_4(s)(\mu_v+\delta_v)u_n\ ds \quad (54)$$

$$\delta v_{n+1}(t) = \delta v_n + \int_0^t \delta\lambda_5(s)\frac{dv_n}{dt} + \delta\lambda_5(s)\mu_v v_n\ ds \quad (55)$$

Thus, we obtain the following stationary conditions

$$\delta x_{n+1} = \delta(1+\lambda_1)x_n + \int_0^t \delta[\lambda'_1 + (\mu_h+p)\lambda_1]x_n\ ds, \quad \mathbf{(56)}$$

$$\delta y_{n+1} = \delta(1+\lambda_2)y_n + \int_0^t \delta[\lambda'_2 + (\mu_h+\varphi_h)\lambda_2]y_n\ ds, \quad (57)$$

$$\delta z_{n+1} = \delta(1+\lambda_3)z_n + \int_0^t \delta[\lambda'_3 - \delta(\mu_h+\gamma_h+\alpha_h)\lambda_3]z_n\ ds, \quad (58)$$

$$\delta u_{n+1} = \delta(1+\lambda_4)u_n + \int_0^t \delta[\lambda'_4 + (\mu_v+\delta_v)\lambda_4]y_n\ ds, \quad (59)$$

$$\delta v_{n+1} = \delta(1+\lambda_5)v_n + \int_0^t \delta[\lambda'_5 + \mu_v\lambda_5]v_n\ ds, \quad (60)$$

Thus, we obtain the following stationary conditions

$$\begin{aligned}
\delta x_n &: (1-\lambda_1(t))|_{s=t} = 0,\\
\delta y_n &: (1-\lambda_2(t))|_{s=t} = 0,\\
\delta z_n &: (1-\lambda_3(t))\big|_{s=t} = 0,\\
\delta u_n &: (1-\lambda_4(t))|_{s=t} = 0\\
\delta v_n &: (1-\lambda_5(t))|_{s=t} = 0\\
\delta x'_n &: \lambda_1{}'(s) + (\mu_h+p)\lambda_1(s)|_{s=t} = 0,\\
\delta y'_n &: \lambda_2(s) + (\mu_h+\varphi_h)\lambda_2(s)|_{s=t} = 0,\\
\delta z'_n &: \lambda_3(s) - (\mu_h+\gamma_h+\alpha_h)\lambda_3(s)|_{s=t} = 0,\\
\delta u'_n &: \lambda_4(s) + (\mu_v+\delta_v)\lambda_4(s)|_{s=t} = 0\\
\delta v'_n &: \lambda_5(s) + \mu_v\lambda_5(s)|_{s=t} = 0
\end{aligned}$$

Solving this system of equations yields

$$\begin{aligned}
\lambda_1(s) &= -e^{(\mu_h+p_1)(s-t)},\\
\lambda_2(s) &= -e^{(\mu_h+\varphi_h)(s-t)},\\
\lambda_3(s) &= -e^{-(\mu_h+\gamma_h+\alpha_h)(s-t)},\\
\lambda_4(s) &= -e^{(\mu_v+\delta_v)(s-t)}\\
\lambda_5(s) &= -e^{\mu_v(s-t)}
\end{aligned} \quad \mathbf{(61)}$$

Here, the general Lagrange multiplier in (88) is expanded by Taylor series and is chosen only one term in order to calculate, the general Lagrange multiplier can write as follows

$$\begin{aligned}
\lambda_1(s) &= -1,\\
\lambda_2(s) &= -1,\\
\lambda_3(s) &= -1,\\
\lambda_4(s) &= -1,\\
\lambda_5(s) &= -1.
\end{aligned} \quad \mathbf{(62)}$$

Substituting the general Lagrange multipliers in (89) into the correction functional in (73)-(75)results in the following iteration formula:

$$x_{n+1}(t) = x_n - \int_0^t \lambda_1\left[\frac{dx_n}{dt} - \mu_h(1-x_n) + p_1x_n - \alpha x_n z_n\right] ds, \quad (63)$$

$$y_{n+1}(t) = y_n - \int_0^t \left[\frac{dy_n}{dt} - (\alpha u_n + p_1)z_n + (\mu_h+\varphi_h)y_n\right] ds, \quad (64)$$

$$z_{n+1}(t) = z_n - \int_0^t \left[\frac{dz_n}{ds} - \varphi_h y_n - (\mu_h+\gamma_h+\alpha_h)z_n\right] ds, \quad (65)$$

$$u_{n+1}(t) = u_n - \int_0^t \left[\frac{du_n}{dt} - \gamma_v(1-v_n-u_n)z_n + (\mu_v+\delta_v)u_n\right] ds \quad (66)$$

$$v_{n+1}(t) = v_n - \int_0^t \left[\frac{dv_n}{dt} - \delta_v u_n + \mu_v v_n\right] ds \quad (67)$$

The iteration starts with an initial approximation as by Health ministry of Indonesia [1], $x_0 = \frac{7675406}{7675893}, y_0 = \frac{76759}{7675893}, z_0 = \frac{487}{7675893}, u_0 = 0.01$and as$v_0 = 0.056$as well as parameters $\alpha = \frac{b\beta_h A}{\mu_v N_h}, \mu_h = 0.000046, p_1 = 0.09, \varphi_h = 0.1667, \gamma_h = 0.3288330, \alpha = 0.0000002, \delta_v = 0.1428000. \mu_v = 0.0323000$, $b\beta_h = 0.75$and $b\beta_v = 0.375$. The iteration formula (90)-(92) now yields

$$\begin{aligned}
x_1 &= \mathbf{0.9999365546 - 0.10129965580}\ t, && (68)\\
y_1 &= 0.01000000912 + 0.1013290994\ t, && (69)\\
z_1 &= 0.00006344538675 + 0.001646135652\, t, && (70)\\
u_1 &= 0.01 - 0.001728778253\, t, && (71)\\
v_1 &= 0.056 - 0.0003808000000\, t, && (72)\\
x_2 &= 0.9999365546 - \mathbf{0.10129965580}\ t
\end{aligned}$$

$$+5.351058817 \times 10^{-3}t^2 -3.035688025 \times 10^{-6}t^3, \quad (73)$$

$$y_2 = 0.01000000912 + 0.1013290994\ t +3.035688025 \times 10^{-5}t^3 -0.01379680090\ t^2, \quad (74)$$

$$z_2 = 6.344538675 \times 10^{-5} +0.001646135652\ t +0.008175090545\ t^2, \quad (75)$$

$$u_2 = 0.01 - 0.001728778253\ t +4.340814966 \times 10^{-7}t^3 +4.396591377\ t^2, \quad (76)$$

$$v_2 = 0.056 - 3.808 \times 10^{-4}\ t -1.172848472 \times 10^{-4}t^2, \quad (77)$$

$$x_3 = 0.9999365546 - \mathbf{0.10129965580}\ t +5.351058817 \times 10^{-3}t^2 -1.777657472 \times 10^{-4}t^3 -5.047418598 \times 10^{-7}t^4 +2.909173958 \times 10^{-8}t^5 -1.377864575 \times 10^{-11}t^6, \quad (78)$$

$$y_3 = 0.01000000912 + 0.1013290994\ t -0.01379680090\ t^2 +9.445374854 \times 10^{-4}t^3 +3.782295613 \times 10^{-7}t^4 -2.909173958 \times 10^{-8}t^5 +1.377864575 \times 10^{-11}t^6, \quad (79)$$

$$z_3 = 6.344538675 \times 10^{-5} +0.001646135652\ t +0.008175090545\ t^2 -0.001662847983\ t^3 +1.265122984 \times 10^{-7}t^4, \quad (80)$$

$$u_3 = 0.01 - 0.001728778253\ t +4.396591377\ t^2 +9.292119076 \times 10^{-4}t^3 +1.548056883 \times 10^{-6}t^4 -1.977115178 \times 10^{-7}t^5 -2.217909712 \times 10^{-10}t^6, \quad (81)$$

$$v_3 = 0.056 - 3.808 \times 10^{-4}\ t -1.172848472 \times 10^{-4}t^2 +2.219054181 \times 10^{-5}t^3 +1.549670943 \times 10^{-8}t^4, \quad (82)$$

and so on.

5. Implementation of HPM

First, write the SEIR model of dengue feverin the following form:

$$\frac{dx}{dt} = \mu_h(1-x) - p_1x + \alpha xz,$$

$$\frac{dy}{dt} = (\alpha u + p_1)z - (\mu_h + \varphi_h)y,$$

$$\frac{dz}{dt} = \varphi_h y - (\mu_h + \gamma_h + \alpha_h)z,$$

$$\frac{du}{dt} = \gamma_v(1-v-u)z - (\mu_v + \delta_v)u$$

$$\frac{dv}{dt} = \delta_v u - \mu_v v$$

subject to the initial conditions

$$\boldsymbol{x(t_0) = c_1,\quad y(t_0) = c_2,\quad z(t_0) = c_3,\quad u(t_0) = c_4,\quad v(t_0) = gc_5,} \quad (83)$$

According to HPM, we construct a homotopy for (1)–(5) which is satisfies the following relations:

$$\boldsymbol{a'_1 - x'_0 + p(x'_0 - \mu_h(1-a_1) + p_1a_1 - \alpha a_1a_3) = 0,} \quad (84)$$

$$\boldsymbol{a'_2 - y'_0 + p(y'_0 - (\alpha a_4 + p_1)a_3 + (\mu_h + \varphi_h)a_2) = 0,} \quad (85)$$

$$\boldsymbol{a'_3 - z'_0 + p(z'_0 - \varphi_h a_2 + (\mu_h + \gamma_h + \alpha_h)a_3) = 0.} \quad (86)$$

$$\boldsymbol{a'_4 - u'_0 + p(u'_0 - \gamma_v(1 - a_5 - a_4)a_3 + (\mu_v + \delta_v)a_4) = 0.} \quad (87)$$

$$\boldsymbol{a'_5 - v'_0 + p(v'_0 - \delta_v a_4 + \mu_v a_5) = 0.} \quad (88)$$

Let us choose the initial approximations as

$$\boldsymbol{a_{1,0}(t) = x_0(t) = a_1(0) = c_1,} \quad (89)$$
$$\boldsymbol{a_{2,0}(t) = y_0(t) = a_2(0) = c_2,} \quad (90)$$
$$\boldsymbol{a_{3,0}(t) = z_0(t) = a_3(0) = c_3,} \quad (91)$$
$$\boldsymbol{a_{4,0}(t) = u_0(t) = a_4(0) = c_4,} \quad (92)$$
$$\boldsymbol{a_{5,0}(t) = v_0(t) = a_5(0) = c_5,} \quad (93)$$

and

$$\boldsymbol{a_1(t) = a_{1,0}(t) + pa_{1,1}(t) + p^2a_{1,2}(t) + p^3a_{1,3}(t) + \cdots,} \quad (94)$$

$$\boldsymbol{a_2(t) = a_{2,0}(t) + pa_{2,1}(t) + p^2a_{2,2}(t) + p^3a_{2,3}(t) + \cdots,} \quad (95)$$

$$\boldsymbol{a_3(t) = a_{3,0}(t) + pa_{3,1}(t) + p^2a_{3,2}(t) + p^3a_{3,3}(t) + \cdots,} \quad (96)$$

$$\boldsymbol{a_4(t) = a_{4,0}(t) + pa_{4,1}(t) + p^2a_{4,2}(t) + p^3a_{4,3}(t) + \cdots,} \quad (97)$$

$$\boldsymbol{a_5(t) = a_{5,0}(t) + pa_{5,1}(t) + p^2a_{5,2}(t) + p^3a_{5,3}(t) + \cdots,} \quad (98)$$

where$a_{i,j}i$ $(i = 1,2;\ j = 1,2,3,\dots)$ are functions yet to be determined. Substituting (94)–(98) into (84)–(88) and collecting terms of the same powers of p, we have

$$\boldsymbol{-0.000046 + 0.090046a_{1,0} + 0.23219814241486064a_{1,0}a_{5,0} + a_{1,1}' = 0,\quad a_{1,1}(0) = 0,} \quad (99)$$

$$\boldsymbol{-0.09a_{1,0} + 0.16674599999999998a_{2,0} - 0.23219814241486064a_{1,0}a_{3,0} + a_{2,1}' = 0,\quad a_{2,1}(0) = 0,} \quad (100)$$

$$\boldsymbol{-0.1667a_{2,0} + 0.3288792a_{3,0} + a_{3,1}' = 0,\quad a_{3,1}(0) = 0,} \quad (101)$$

$$\boldsymbol{-0.375a_{3,0} + 0.1751a_{4,0} + 0.375a_{3,0}a_{4,0} + 0.375a_{3,0}a_{5,0} + a_{4,1}' = 0,\quad a_{4,1}(0) = 0,} \quad (102)$$

$$-0.1428a_{4,0}+0.0323a_{5,0}+a_{5,1}'=0,\quad a_{5,1}(0)=0 \tag{103}$$

$$0.090046a_{1,1}+0.23219814241486064a_{1,1}a_{5,0}+0.23219814241486064a_{1,0}a_{5,1}+a_{1,2}'=0,\quad a_{1,2}(0)=0 \tag{104}$$

$$-0.09a_{1,1}+0.16674599999999998a_{2,1}-0.23219814241486064a_{1,1}a_{3,0}-0.23219814241486064a_{1,0}a_{3,1}+a_{2,2}'=0,,\quad a_{2,2}(0)=0 \tag{105}$$

$$-0.1667a_{2,1}+0.3288792a_{3,1}+a_{3,2}'=0,\quad a_{3,2}(0)=0, \tag{106}$$

$$-0.375a_{3,1}+0.375a_{3,1}a_{4,0}+0.1751a_{4,1}+0.375a_{3,0}a_{4,1}+0.375a_{3,1}a_{5,0}+0.375a_{3,0}a_{5,1}+a_{4,2}'=0,\quad a_{4,2}(0)=0 \tag{107}$$

$$-0.1428a_{4,1}+0.0323a_{5,1}+a_{5,2}',\quad a_{5,2}(0)=0, \tag{108}$$

$$0.090046a_{1,2}+0.23219814241486064a_{1,2}a_{5,0}+0.23219814241486064a_{1,1}a_{5,1}+0.23219814241486064a_{1,0}a_{5,2}+a_{1,3}'\quad a_{1,3}(0)=0, \tag{109}$$

$$-0.09a_{1,2}+0.16674599999999998a_{2,2}-0.23219814241486064a_{1,2}a_{3,0}-0.23219814241486064a_{1,1}a_{3,1}-0.23219814241486064a_{1,0}a_{3,2}+a_{2,3}'\quad a_{2,3}(0)=0, \tag{110}$$

$$-0.1667a_{2,2}+0.3288792a_{3,2}+a_{3,3}',\quad a_{3,3}(0)=0, \tag{111}$$

$$-0.375a_{3,2}+0.375c_2a_{4,0}+0.375a_{3,1}a_{4,1}+0.1751a_{4,2}+0.375a_{3,0}a_{4,2}+0.375a_{3,2}a_{5,0}+0.375a_{3,1}a_{5,1}+0.375a_{3,0}a_{5,2}+a_{4,3}'=0,\quad a_{4,3}(0)=0, \tag{112}$$

$$-0.1428a_{4,2}+0.0323a_{5,2}+a_{5,3}'=0,\quad a_{5,3}(0)=0, \tag{113}$$

Solving the differential equations (99)–(113) we get,

$$a_{1,1}=\int_0^t\left[0.000046-0.090046a_{1,0}+0.23219814241486064a_{1,0}a_{5,0}\right]ds, \tag{114}$$

$$a_{2,1}=\int_0^t\left[0.09a_{1,0}-0.16674599999999998a_{2,0}+0.23219814241486064a_{1,0}a_{3,0}\right]ds, \tag{115}$$

$$a_{3,1}=\int_0^t\left[0.375a_{3,0}-0.1751a_{4,0}-0.375a_{3,0}a_{4,0}-0.375a_{3,0}a_{5,0}\right]ds, \tag{116}$$

$$a_{4,1}=\int_0^t\left[0.375a_{3,0}-0.1751a_{4,0}-0.375a_{3,0}a_{4,0}-0.375a_{3,0}a_{5,0}\right]ds \tag{117}$$

$$a_{5,1}=\int_0^t\left[0.1428a_{4,0}-0.0323a_{5,0}\right]ds \tag{118}$$

$$a_{1,2}=\int_0^t\left[-0.090046a_{1,1}-0.23219814241486064a_{1,1}a_{5,0}-0.23219814241486064a_{1,0}a_{5,1}\right]ds, \tag{119}$$

$$a_{2,2}=\int_0^t\left[0.09a_{1,1}-0.16674599999999998a_{2,1}+0.23219814241486064a_{1,1}a_{3,0}+0.23219814241486064a_{1,0}a_{3,1}\right]ds, \tag{120}$$

$$a_{3,2}=\int_0^t\left[0.1667a_{2,1}-0.3288792a_{3,1}\right]ds, \tag{121}$$

$$a_{4,2}=\int_0^t\left[0.375a_{3,1}-0.375a_{3,1}a_{4,0}-0.1751a_{4,1}-0.375a_{3,0}a_{4,1}-0.375a_{3,1}a_{5,0}-0.375a_{3,0}a_{5,1}\right]ds, \tag{122}$$

$$a_{5,2}=\int_0^t\left[0.1428a_{4,1}-0.0323a_{5,1}\right]ds, \tag{123}$$

$$a_{1,3}=\int_0^t\left[-0.090046a_{1,2}-0.23219814241486064a_{1,2}a_{5,0}-0.23219814241486064a_{1,1}a_{5,1}-0.23219814241486064a_{1,0}a_{5,2}\right]ds, \tag{124}$$

$$a_{2,3}=\int_0^t\left[0.09a_{1,2}-0.16674599999999998a_{2,2}+0.23219814241486064a_{1,2}a_{3,0}+0.23219814241486064a_{1,1}a_{3,1}+0.23219814241486064a_{1,0}a_{3,2}\right]ds, \tag{125}$$

$$a_{3,3}=\int_0^t\left[0.1667a_{2,2}-0.3288792a_{3,2}\right]ds, \tag{126}$$

$$a_{4,3}=\int_0^t\left[0.375a_{3,2}-0.375c_2a_{4,0}-0.375a_{3,1}a_{4,1}-0.1751a_{4,2}-0.375a_{3,0}a_{4,2}-0.375a_{3,2}a_{5,0}-0.375a_{3,1}a_{5,1}-0.375a_{3,0}a_{5,2}\right]ds, \tag{127}$$

$$a_{5,3}=\int_0^t\left[0.1428a_{4,2}-0.0323a_{5,2}\right]ds, \tag{128}$$

Taking the actual physiological data from Health Ministry of Indonesia[20], $c_1 = \frac{7675406}{7675893}, c_2 = \frac{487}{7675893}, c_3 = 0.056$ as well as $\alpha = 0.232198, \beta = 0.328879, \gamma = 0.375,$ and $\delta_1 = 0.0323$ yields

$\boldsymbol{a_{1,1}}$	$\boldsymbol{-0.10299655798548431t}$	(129)
$\boldsymbol{a_{2,1}}$	$0.08834155936083532t$	(130)
$\boldsymbol{a_{3,1}}$	$0.001646135652177538t$	(131)
$\boldsymbol{a_{4,1}}$	$-0.0017287782532924836t$	(132)
$\boldsymbol{a_{5,1}}$	$-0.00038079999999999993t$	(133)
$\boldsymbol{a_{1,2}}$	$0.005351058815844219t^2$	(134)
$\boldsymbol{a_{2,2}}$	$-0.011809801910537217t^2$	(135)
$\boldsymbol{a_{3,2}}$	$0.00709257908453581t^2$	(136)
$\boldsymbol{a_{4,2}}$	$0.00043965913772737634t^2$	(137)
$\boldsymbol{a_{5,2}}$	$-0.00011728484728508333t^2$	(138)
$\boldsymbol{a_{1,3}}$	$-0.00017776574723776502t^3$	(139)
$\boldsymbol{a_{2,3}}$	$0.0013527740726381188t^3$	(140)
$\boldsymbol{a_{3,3}}$	$-0.0014337652379151412t^3$	(141)
$\boldsymbol{a_{4,3}}$	$0.0008028286946324015t^3$	(142)
$\boldsymbol{a_{5,3}}$	$.0000221905t^3$	(143)

The 11th-term HPM solutions,

$$\boldsymbol{x(t) = \sum_{j=0}^{10} a_{1,j} = \frac{7675406}{7675893} - 0.10299655798548431t + 0.005351058815844219t^2 + \cdots,} \tag{144}$$

$$\boldsymbol{y(t) = \sum_{j=0}^{10} a_{2,j} = \frac{76759}{7675893} + 0.08834155936083532t - 0.011809801910537217t^2 + \cdots,} \tag{145}$$

$$\boldsymbol{z(t) = \sum_{j=0}^{10} a_{4,j} = \frac{487}{7675893} + 0.001646135652177538t + 0.00709257908453581t^2 + \cdots,} \tag{146}$$

$$\boldsymbol{u(t) = \sum_{j=0}^{10} a_{3,j} = 0.01 - 0.0017287782532924836t + 0.00043965913772737634t^2 + \cdots,} \tag{147}$$

$$\boldsymbol{v(t) = \sum_{j=0}^{10} a_{5,j}} = 0.056 \boldsymbol{- 0.00038079999999999993t - 0.00011728484728508333t^2 + \cdots.,} \tag{148}$$

In this paper, we calculated the HPM until tenth term to obtain the reliable solution. It can be calculated for more terms to reach the exact solution.

6. Result and Discussions

Susceptible exposed infected ad recovery model (SEIR) was solved. From the data in [12], some parameters(γ_h)=0.3288330, $(b\beta_v)$ =0.3750000$(b\beta_h)$ = 0.7500000, (μ_h) = 0.0000460, p_1=0.09 and (μ_v) =0.0323000. The iteration and term was start by$x(0) = \frac{7675406}{7675893}$, $y(0) = \frac{76759}{7675893}$ $z(0) = \frac{487}{7675893}$, $u(0) = 0.01$and $v(0) = 0.056$ .The iterative system of SEIR model was coded in the Maple package by restricting the number of significant Digits in its environment to 16. We then display the comparisons between RK4 solution and collected data in [12], see figure 1. From figure 1, RK4 solution with $\Delta t = 0.001$ is exactly same as plotting data that showed in [12]. Thus RK4 solutions are bachmark of this problem. Figure 2 present VIM, HPM and RK4 solutions with $\Delta t = 0.001$for $t \in [0, 12]$. From figure 2, the 11th iterate of VIM is more accurate than 11th term of HPM for long interval. VIM solutions converge to RK4 solution and plotting of collected data [12] at certain times. VIM also is easier in calculation when it compares to HPM, see in Table 1.1 and Table 1.2.Moreover,Both the VIM and RK4 solutions showed good synchronization at the time performed and both the results agree very well with each other.

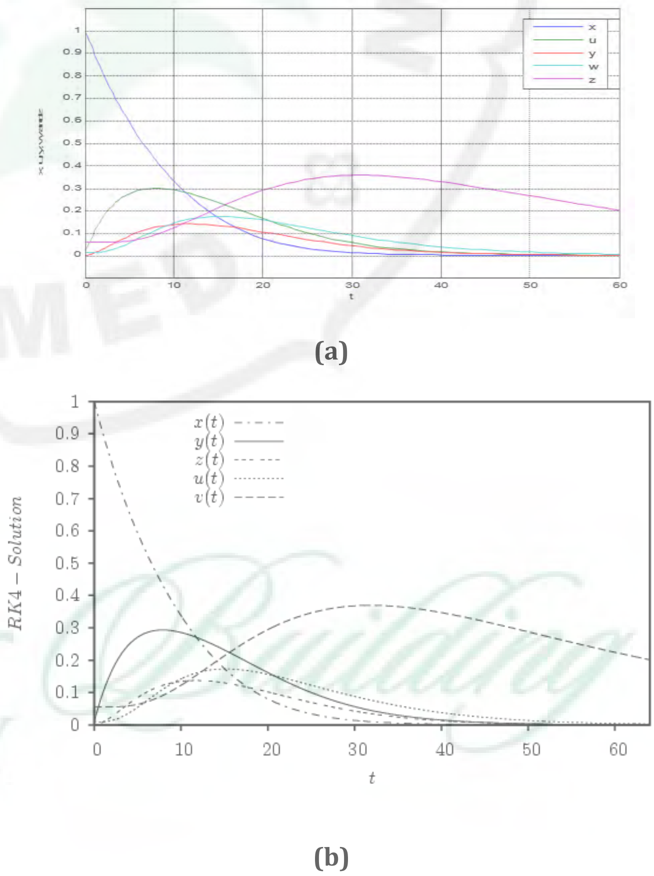
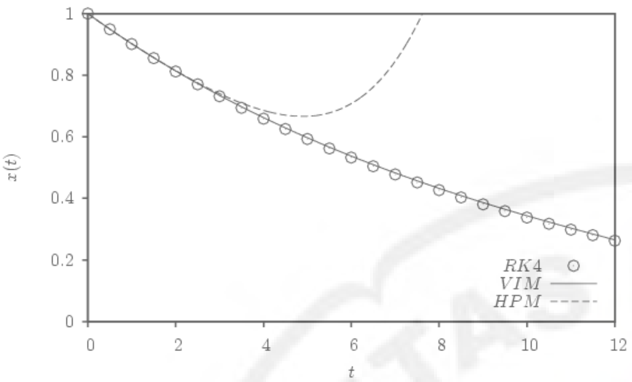
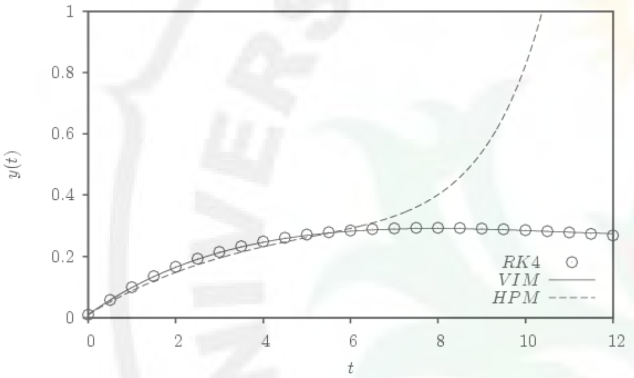


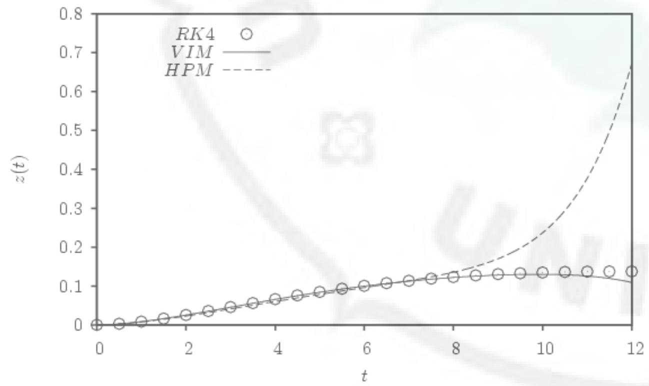
Figure 1. The succeptible (x(t)), infected (y(t)) and Removed/ Recovery (z(t)) populations using (a) RK4 for ⎾t =0.001 and (b) ODESOLVE [1]



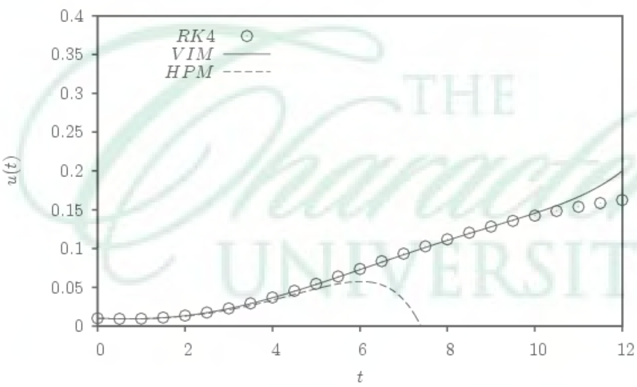
(a)



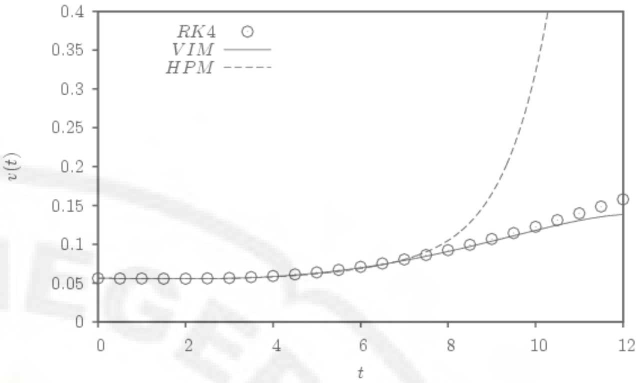
(b)



(c)



(d)



(e)

Figure 2. Approximate solution of (a) susceptible population, (b) infected population and (c) (d) and (e) Vector population using: RK4 for Δt =0.001, 11 terms of HPM, and 11 iterate of VIM, respectively.

Table 1.1. The error of 11st iterate of VIM when it compares to RK4 with Δt =0.001

t	VIM				
	Δx	Δy	Δz	Δu	Δv
0.0	0	0	0	0	0
0.5	6.101 E-04	5.85 E-04	2.377E-05	1.841E-05	7.700E-05
1.0	1.165 E-03	1.07 E-03	8.489E-05	4.202E-05	1.541 E-04
1.5	1.669 E-03	1.468 E-03	1.705 E-04	7.696E-05	2.315 E-04
2.0	2.126 E-03	1.789 E-03	2.706 E-04	1.272 E-04	3.099 E-04
2.5	2.541 E-03	2.043 E-03	3.775 E-04	1.949 E-04	3.901 E-04
3.0	2.919 E-03	2.242 E-03	4.855 E-04	2.807 E-04	4.732 E-04
3.5	3.265 E-03	2.395 E-03	5.905 E-04	3.84 E-04	5.604 E-04
4.0	3.584 E-03	2.51 E-03	6.899 E-04	5.032 E-04	6.532 E-04
4.5	3.881 E-03	2.594 E-03	7.819E-04	6.36 E-04	7.528 E-04
5.0	4.161 E-03	2.654 E-03	8.662E-04	7.792 E-04	8.609 E-04

Table 1.2. The error of 11st term of HPM when it compares to RK4 with $\Delta t = 0.001$

t	HPM				
	Δx	Δy	Δz	Δu	Δv
0.0	0	0	0	0	0
0.5	6.109E-04	6.539E-03	2.66E-04	3.265E-05	7.726E-05
1.0	1.189E-03	1.179E-02	9.44E-04	1.441E-04	1.579E-04
1.5	1.849E-03	1.569E-02	1.87E-03	3.835E-04	2.49E-04
2.0	2.878E-03	1.829E-02	2.903E-03	7.697E-04	3.60E-04
2.5	4.815E-03	1.966 E-02	3.93E-03	1.298E-03	5.006E-04
3.0	8.529E-03	1.991E-02	4.862E-03	1.947E-03	6.798E-04
3.5	1.528E-02	1.909E-02	5.629E-03	2.699E-03	9.043E-04
4.0	2.682E-02	1.719E-02	6.174E-03	3.568E-03	1.177E-03
4.5	4.539E-02	1.408E-02	6.441E-03	4.67E-03	1.491E-03
5.0	7.385E-02	9.461E-03	6.365E-03	6.379E-03	1.823E-03

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