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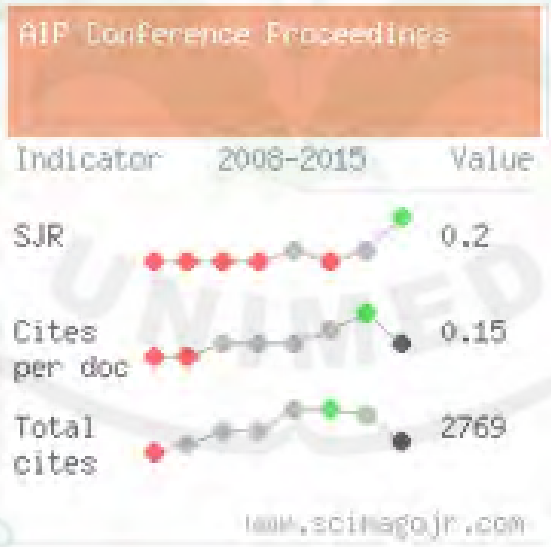

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A VSEIR model for transmission of tuberculosis (TB) disease in North Sumatera, Indonesia

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A VSEIR Model for Transmission of Tuberculosis (TB) Disease in North Sumatera, Indonesia

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Abstract. In this work, Vaccination (V), Susceptible (S), Infected (I), and Recovered (R) (VSIR) model for transmission of Tuberculosis in North Sumatera is modified. An exposed class is adopted to VSIR model so called VSEIR to determine the probability of people who infectious before infected. This model is written in ordinary differential equation (ODEs) in five classes. Determination the equilibrium point and stability analysis of the model is discussed to determine the dynamic behaviour of systems. A simulation is also discussed to see the suitable model to North Sumatera data. The simulation of VSEIR model indicates Tuberculosis has not endemic in North Sumatera

Keywords*:* VSEIR model; Stability analysis; equilibrium analysis; Tuberculois
PACS: 6460Ht; 8723Cc; 82.60.Hc

INTRODUCTION

Tuberculosis (TB) is a bacterial disease acquired through airborne infection. Mycobacterium tuberculosis (MTB) is the causative agent of tuberculosis. TB disease can affect anyone (old, young, men, women, poor, or rich) and anywhere. TB disease is usually transmitted through contaminated air with Mycobacterium tuberculosis bacteria that are released during coughing TB patients, and in children the source of infection is generally derived from adult TB patients. These bacteria often enter and when accumulated in the lungs will breed a lot (especially in people with a low immune system), and can spread through the blood vessels or lymph nodes. That is why TB infection can infect virtually all body organs such as the lungs, brain, kidneys, gastrointestinal tract, bone, lymph nodes, etc., although the organs most commonly affected are the lungs [1]. Each year, Indonesia increased by a quarter of a million new TB cases and approximately 140,000 deaths occur each year due to tuberculosis. In fact, Indonesia is the third largest country with the problem of tuberculosis in the world [1]. According to the World Health Organization, one –third the world's population is infected, either latently or actively with tuberculosis [2].

During the year 2010, around 73.8 percent of TB patients are in North Sumatra. Based on a survey of these, Medan city is the largest number of sufferers. In general, the detection rate of TB case increased in North Sumatra. According to the North Sumatra Department of Health in 2005, we estimated that at 15,517 cases of TB sufferers and in 2010 as many as 15,614 TB-positive people in North Sumatra, while based on the estimated, it amounts to 21 148 people. Based on data from the Department of Health in 2010 there are six districts/ cities in North Sumatra in 2010 with the highest number of patients based on the population in Medan around 2,397 patients, Siantar around 288 patients, Binjai around 260 patients, Tanjung Balai around 150 patients, Tebing Tinggi around 145 patients and Deli Serdang around 1,554 patients [3].

Immunization is considered important because it has some benefits for toddlers, such as preventing the spread of Tuberculosis. BCG immunization was given 1 month of age, giving one the benefit prevent transmission of tuberculosis (TB) are heavy. "If the baby is not completely immunized under the age of one year as BCG has not given, it must be done if the test maontoux baby five months of age or older. This test is to determine whether the baby is negatively affected by TB. If the test result is negative, can only be given BCG immunization [1].

The mathematical model for tuberculosis found that compartmental dynamics such as Susceptible, Infected, Removed with vaccination (VSIR) [4]. Since the disease can remain latent, become active, or it can progress from latent TB to active TB either by endogenous reactivation or exogenous reinfection [5]. Based on the previous statement, we modify [4] and adopts the class Exposed (E) to VSIR model. Thus, this paper will discuss about formuation of model, analysis and simulation using the fourth order Runge Kutta (KR4).

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MODEL FORMULATION

The total population size $N(t)$ is divided into four distinct epidemiological subclasses of individuals which are vavaccinatedsusceptible, ininfectedand recovered, with sizes denoted by $V(t), S(t)$, $I(t)$, and $R(t)$, respectively. Thus, $N(t)$ can be written as $N(t) = V(t) + S(t) + I(t) + R(t)$. The VSIR model [4] having vaccination, infected and recovered period is described by the following system of differential equations:

$$\frac{dV}{dt} = q - (\mu_1 + \delta_1)V, \tag{1}$$
$$\frac{dS}{dt} = \delta_1 V - (\mu_2 + \delta_2 I)S + \theta S, \tag{2}$$
$$\frac{dI}{dt} = \delta_2 IS - (\mu_4 + \mu_{TB} + \delta_4)I, \tag{3}$$
$$\frac{dR}{dt} = \delta_4 I - (\mu_5 + \theta)R, \tag{4}$$

where human birth in natural through passive vaccination ($V(t)$) at rate p, non negative parameters $\mu_1, \mu_1, \mu_3, \mu_2, \mu_5$ denote as natural death of population of the V, the S, the I and the R, respectively. Population of infected Tuberculosis died in rate μ_{TB}. The susceptible population decreased due to coming individual from the V in rate δ_1. δ_2 denotes the transfer rate from susceptible to infected population. Infected population increases due to movement of individuals from infected individuls I in rate δ_4 dan decreased due to movement of individuals in to the S at rate θ. In this paper, we assume that human recovering is fully recovered. In flow of mathematical model, we assume that each compartment occurs interaction between classes. Hence, Eqs (1)-(4) can be written as

$$\frac{dV}{dt} = qN - \mu_1 V - \delta_1 VS, \tag{5}$$
$$\frac{dS}{dt} = \delta_1 VS - \mu_2 S - \delta_2 IS, \tag{6}$$
$$\frac{dI}{dt} = \delta_2 SI - (\mu_4 + \mu_m + \delta_4)I, \tag{7}$$
$$\frac{dR}{dt} = \delta_4 I - \mu_5 R. \tag{8}$$

Here, we assume that all new birth got BCG vaccination. Using a compartmental approach, one may assume that a susceptible individual first goes through a latent period (and is said to become exposed or in class $E(t)$). The exposed individual increases from susceptible individuals in at rate α and decreases in rate ρ and μ_3 couse of death. Then, any interaction between exposed and infected in rate ρ. The exposed population The VSEIR model having infectious force, infected and recovered period is described by the following system of differential equations:

$$\frac{dV}{dt} = qN - \mu_1 V - \delta_1 VS, \tag{9}$$
$$\frac{dS}{dt} = \delta_1 VS - \mu_2 S - \alpha ES, \tag{10}$$
$$\frac{dE}{dt} = (\alpha S - \mu_3 - \rho I)E, \tag{11}$$
$$\frac{dI}{dt} = \rho EI - (\mu_4 + \mu_{TB} + \delta_4)I, \tag{12}$$
$$\frac{dR}{dt} = \delta_4 I - \mu_5 R, \tag{13}$$

with conditions

$$N = V + S + E + I + R \rightarrow R = N - V - S - I, \tag{14}$$

where the positive parameters $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$ and μ_5 are the rate of natural death of vaccitaion individual ($V(t)$), susceptible individual ($S(t)$), exposed individual ($E(t)$), Infected individual ($I(t)$) and recovery individual ($R(t)$), respectively. q denote the rate of natural birth through passive vaccination. The model can be simplified by assuming the following fractions [6]

$$u = \frac{V}{N}, \; w = \frac{S}{N}, \; x = \frac{E}{N}, \; y = \frac{I}{N}, \text{ and } z = \frac{R}{N}. \tag{15}$$

Thus, the model for human populations can be simplified as follows

$$\frac{du}{dt} = q - (\mu_1 + \delta_1)u, \tag{16}$$
$$\frac{dw}{dt} = (\delta_1 u - \mu_2 - \alpha x)w, \tag{17}$$
$$\frac{dx}{dt} = (\alpha w - \mu_3 - \rho y)x, \tag{18}$$

$$\frac{dy}{dt} = \rho xy - \beta y, \tag{19}$$
$$\frac{dz}{dt} = \delta_4 y - \mu_5 z, \tag{20}$$

where $\beta = (\mu_4 + \mu_{TB} + \delta_4)$.

STABILITY ANALYSIS

Disease Free Equilibrium (DFE)

Critcal point will occur while the value of

$$\frac{du}{dt} = \frac{dw}{dt} = \frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0. \tag{21}$$

Substitute (16)-(20) int to Eq. (21) as follows

$$q - (\mu_1 + \delta_1 w)u = 0, \tag{22}$$
$$\delta_1 uw - (\mu_2 + \alpha x)w = 0, \tag{23}$$
$$(\alpha w - \mu_3 - \rho y)x = 0, \tag{24}$$
$$\rho xy - \beta y = 0, \tag{25}$$
$$\delta_4 y - \mu_5 z = 0. \tag{26}$$

Inserting Eqs. (21)-(25) into Eq. (26) indicates the equilibrium point of the system are: $F_1 = \left(\frac{q}{\mu_1}, 0, 0, 0, 0\right)$, and $F_2 = (u_0, w_0, x_0, y_0, z_0)$ with values

$$u_0 = \frac{\mu_2}{\delta_1}, \qquad w_0 = \frac{q\delta_1 - \mu_1\mu_2}{\delta_1\mu_2}, \quad x_0 = 0, \qquad y_0 = 0, \qquad z_0 = 0. \tag{27}$$

Linearization of Eqs. (16)-(20) on the equilibrium points $\left(\frac{q}{\mu_1}, 0, 0, 0, 0\right)$, yields the following equation

$$\begin{pmatrix} \frac{du}{dt} \\ \frac{dw}{dt} \\ \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix} = \begin{pmatrix} -\mu_1 & \frac{q\alpha}{\mu_1} & 0 & 0 & 0 \\ 0 & \frac{q\delta_1}{\mu_1} - \mu_2 & 0 & 0 & 0 \\ 0 & 0 & -\mu_3 & 0 & 0 \\ 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & \delta_4 & -\mu_5 \end{pmatrix} \begin{pmatrix} u \\ w \\ x \\ y \\ z \end{pmatrix}. \tag{28}$$

Using MAPLE, Eq (28) leads to five eigenvalue equations as follows:

$$\begin{aligned} -\lambda^5 + \left(\frac{q\delta_1}{\mu_1} - \beta - \mu_1 - \mu_2 - \mu_3 - \mu_5\right)\lambda^4 \\ + \left[\left(\frac{q\delta_1(\beta + \mu_3 + \mu_5)}{\mu_1}\right)\right. \\ \left. + (q\delta_1 - (\mu_1 + \mu_5 + \mu_2 + \mu_3)\beta - \mu_3(\mu_1 + \mu_2) - (\mu_5 + \mu_1)\mu_2)\right]\lambda^3 \\ + \left[\frac{q\delta_1(\mu_3\beta + \beta\mu_5 + \mu_5\mu_3)}{\mu_1}\right]\lambda^2 \\ + \left[\frac{q\delta_1\beta(\mu_5 - \mu_3)}{\mu_1} + q\delta_1(\mu_5 + \mu_3 + \beta) - \mu_1\mu_2(\beta + \mu_5 + \mu_3) - \mu_1\mu_5(\beta + \mu_3)\right. \\ \left. - \mu_3\mu_5(\beta + \mu_2) - \mu_2\mu_5(\beta + \mu_3) - \mu_2\mu_3(\beta + \mu_5)\right]\lambda - \mu_1\mu_2\mu_3\mu_5\beta + q\delta_1\mu_3\mu_5\beta = 0, \end{aligned} \tag{29}$$

with eigen values

$$\lambda_1 = -\mu_1, \qquad \lambda_2 = -\frac{\mu_1\mu_2 - q\delta_1}{\mu_1}, \qquad \lambda_3 = -\mu_3, \qquad \lambda_4 = -\beta, \qquad \lambda_5 = -\mu_5. \tag{30}$$

Epidemic Equilibrium State

Linearization of Eqs (16)-(20) on the equilibrium point $(u_0, w_0, x_0, y_0, z_0)$ yields the following equation:

$$\begin{pmatrix} \frac{du}{dt} \\ \frac{dw}{dt} \\ \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix} = \begin{pmatrix} -\frac{q\delta_1 - \mu_1\mu_2}{\mu_2} - \mu_1 & -\mu_2 & 0 & 0 & 0 \\ \frac{q\delta_1 - \mu_1\mu_2}{\mu_2} & 0 & -\frac{\delta_2(q\delta_1 - \mu_1\mu_2)}{\delta_1\mu_2} & 0 & 0 \\ 0 & 0 & \frac{\alpha(q\delta_1 - \mu_1\mu_2)}{\delta_1\mu_2} - \mu_3 & 0 & 0 \\ 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & \delta_4 & -\mu_5 \end{pmatrix} \begin{pmatrix} u \\ w \\ x \\ y \\ z \end{pmatrix}. \tag{31}$$

Using MAPLE, Eq (28) leads to five eigenvalue equations as follows

$$\begin{aligned} -\lambda^5 + \left[-\beta - \mu_3 - \mu_5 - \frac{q(\alpha - \delta_1)}{\mu_2} - \frac{\mu_1\alpha}{\delta_1}\right]\lambda^4 \\ + \left[-(\mu_5 + \mu_3)\beta - q\delta_1 + \mu_1\mu_2 - \mu_3\mu_5 - \frac{\alpha\mu_1\beta}{\delta_1} + \frac{q\alpha\beta}{\mu_2} - \frac{q\delta_1\mu_5}{\mu_2} + \frac{\alpha q^2\delta_1}{\mu_2^2} - \frac{q\delta_1\mu_3}{\mu_2}\right. \\ \left. + \frac{q\alpha\mu_5}{\mu_2} - \frac{q\alpha\mu_1}{\mu_2}\right]\lambda^3 \\ + \left[\frac{\delta_1 q^2\alpha}{\mu_2} + \frac{\mu_2\alpha\mu_1^2}{\delta_1} + \frac{\delta_1 q^2\alpha\beta}{\mu_2^2} + \frac{\delta_1 q^2\alpha\mu_5}{\mu_2^2} - \frac{q\delta_1\beta(\mu_3 + \mu_5)}{\mu_2} - \frac{q\alpha\beta(\mu_1 + \mu_5)}{\mu_2}\right. \\ \left. - \frac{\alpha\mu_1\mu_5\beta}{\delta_1} - \delta_1 q(\mu_5 - \mu_3) + \mu_2\mu_1(\mu_5 + \mu_3) - \delta_1 q\beta + (\mu_2\mu_1 - \mu_3\mu_5)\beta - 2q\alpha\mu_1\right]\lambda^2 \\ + \left[\frac{\delta_1\alpha q^2\beta}{\mu_2} + \frac{\delta_1\alpha q^2\mu_5}{\mu_2} + \frac{\delta_1\alpha q^2\beta\mu_5}{{\mu_2}^2} + \frac{\mu_2\alpha{\mu_1}^2\beta}{\delta_1} + \frac{\mu_2\alpha{\mu_1}^2\mu_5}{\delta_1} - \frac{\delta_1 q\beta\mu_5(\mu_1 + \mu_3)}{\mu_2}\right. \\ \left. - \delta_1 q\beta(\mu_3 + \mu_5) - \delta_1 q\mu_3\mu_5 + \mu_2\mu_1\beta(\mu_3 + \mu_5) + \mu_2\mu_1\mu_3\mu_5 - 2q\alpha\mu_1(\beta - \mu_5)\right]\lambda \\ + \left[\frac{\delta_1 q^2\alpha\mu_5\beta}{\mu_2} + \frac{\mu_2\alpha\mu_1^2\beta\mu_5}{\delta_1} - \delta_1 q\beta\mu_3\mu_5 + \beta\mu_1\mu_2\mu_3\mu_5 - 2q\alpha\beta\mu_1\mu_5\right] = 0 \end{aligned} \tag{32}$$

with eigenvalues

$$\lambda_1 = -\frac{q\delta_1 - \sqrt{q^2\delta_1^2 - 4q\delta_1\mu_2^2 + 4\mu_1\mu_2^3}}{2\mu_2}, \quad \lambda_2 = -\frac{q\delta_1 + \sqrt{q^2\delta_1^2 - 4q\delta_1\mu_2^2 + 4\mu_1\mu_2^3}}{2\mu_2},$$

$$\lambda_3 = -\frac{\delta_1\mu_2\mu_3 - q\delta_1\alpha + \alpha\mu_1\mu_2}{\mu_2\delta_1}, \quad \lambda_4 = -\beta, \quad \lambda_5 = -\mu_5. \tag{33}$$

Since $R_e(\lambda_1) < 0$, $R_e(\lambda_2) < 0, R_e(\lambda_3) < 0, R_e(\lambda_4) < 0$, $R_e(\lambda_5) < 0$, then it is asymptomatically stable.

Equilibrium Point of VSEIR Model for North Sumatera Indonesia

Parameters of this model are variously determined. Some parameters are took from annualy Health fact [1] and supplement data from previous study by Momoh et al. [4]. The parameter is known as $q = 0.11, \mu_1 = 0.1, \mu_2 = 0.133, \mu_3 = 0.14, \mu_5 = 0.133, \delta_1 = 0.675, \delta_2 = 0.544, \delta_3 = 0.644, \delta_4 = 0.7$ and $\mu_{TB} = 0.05$. The equilibrium points were determined using VSEIR model with set parameters for the state of north Sumatera.

$$\frac{du}{dt} = 0.15 - 0.1u - 0.675uw, \tag{34}$$

$$\frac{dw}{dt} = 0.675\,uw - 0.03w - 0.544wx, \tag{35}$$

$$\frac{dx}{dt} = 0.544wx - 0.04x - 0.644xy, \quad (36)$$
$$\frac{dy}{dt} = 0.644xy - 0.904y, \quad (37)$$
$$\frac{dz}{dt} = 0.7y - 0.133z. \quad (38)$$

Then we obtain the critical point defined by $\frac{du}{dt} = \frac{dw}{dt} = \frac{dx}{dt} = \frac{dy}{dt} = \frac{dz}{dt} = 0$, as below

$$0.15 - 0.1u - 0.675uw = 0, \quad (39)$$
$$0.675\,uw - 0.03w - 0.544wx = 0, \quad (40)$$
$$0.544wx - 0.04x - 0.644xy = 0, \quad (41)$$
$$0.644xy - 0.904y = 0, \quad (42)$$
$$0.7y - 0.133z = 0. \quad (43)$$

The equilibrium point of VSEIR model are

$$(u, w, x, y, z) = (V, S, E, I, R) = (1.11,0,0,0,0) \quad (44)$$

and

$$(u, w, x, y, z) = (0.197037037, 0.686438318, 0, 0\ ,0). \quad (45)$$

The second equilibrium points is $(0.197037037, 0.686438318, 0, 0\ ,0)$,whereas, other points are not logic for equilibrium points because any negative point indeed. By using MAPLE, the eigenvalue (λ) Are investigated; as follows: at the equilibrium point $(1.11,0,0,0,0)$, eigen values $\lambda_1 = 0.61625, \lambda_2 = -0.904, \lambda_3 = -0.1, \lambda_4 = -0.14$ and $\lambda_5 = -0.133$. At equilibrium point $(0.197037037, 0.686438318, 0, 0\ ,0)$ has eigenvalue, such as $\lambda_1 = 0.233422445, \lambda_2 = -0.1329999996, \lambda_3 = -0.1485765736, \lambda_4 = -0.4147692913$ and $\lambda_5 = -0.9040000006$.

VSEIR Model for Stability Analysis in North Sumatera Indonesia

Result of VSEIR model in searching the equilibrium point and eigenvalues are discussed in Table 1. Based on the table, The equilibrium points of VSEIR model in North Sumatera is saddle points. It indicates that no occurance of infected Tuberculosis since there are no infected human when 1.11 human are suspected of TB. Every human in the population are health and there aren't human that infected by virus.

Table (1). Equilibrium points and Stability Analysis

Equilibrium points (V, S, E, I, R)	Eigen values	Stability analysis
$(1.11,0,0,0,0)$	Real and opposite sign	Saddle point
$(0.197037037, 0.686438318, 0, 0\ ,0)$	Real and opposite sign	Saddle point

RESULT AND DISCUSSION

VSEIR Model of Tuberculosis in North Sumatera

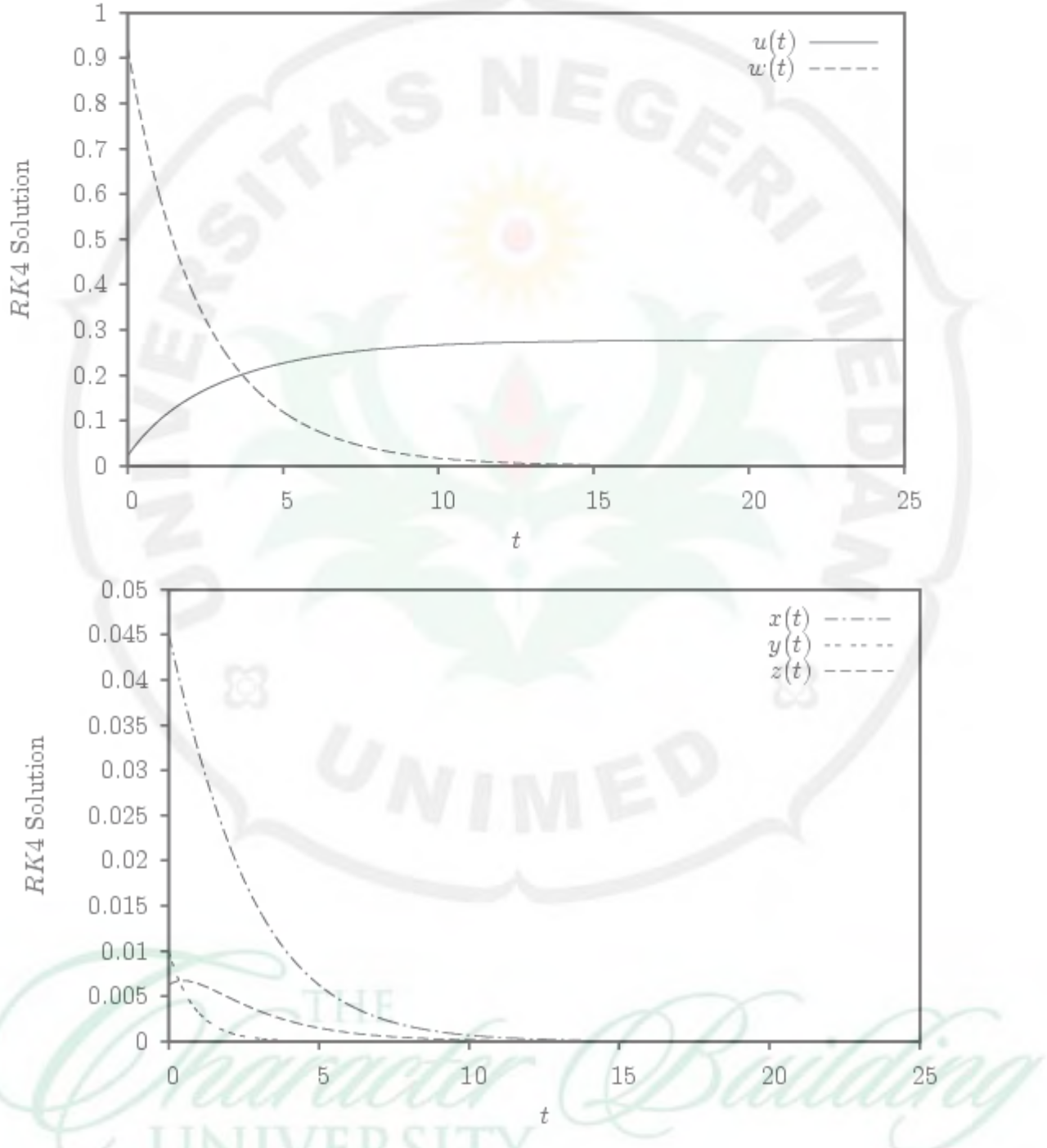
Several investigation have done for VSEIR model of Tuberculosis in this paper. This model is suitable for the state of North Sumatera. Some parameters are took from annualy Health fact [1] and supplement data from previous study by Momoh et al. [4]. The parameter is known as $q = 0.11, \mu_1 = 0.1, \mu_2 = 0.133, \mu_3 = 0.14, \mu_5 = 0.133, \delta_1 = 0.675, \alpha = 0.544, \rho = 0.644, \delta_4 = 0.7$ and $\mu_{TB} = 0.05$. The initial polulation is reported by health department of North Sumatera[3]. Table 1 show the stability analysis finding from equilibrium and eigen valus. From table, all equilibrium points were saddle point. Determining a breeding rate on VSEIR is important in Epidemiology problem since this rate shows the infected population will occur in main state. The determination of R_0 was proposed by [7] where

$$R_0 = \frac{q\delta_1\beta - \mu_1\mu_2\mu_4}{\mu_1\mu_2(\delta_4 + \mu_{TB})}. \quad (46)$$

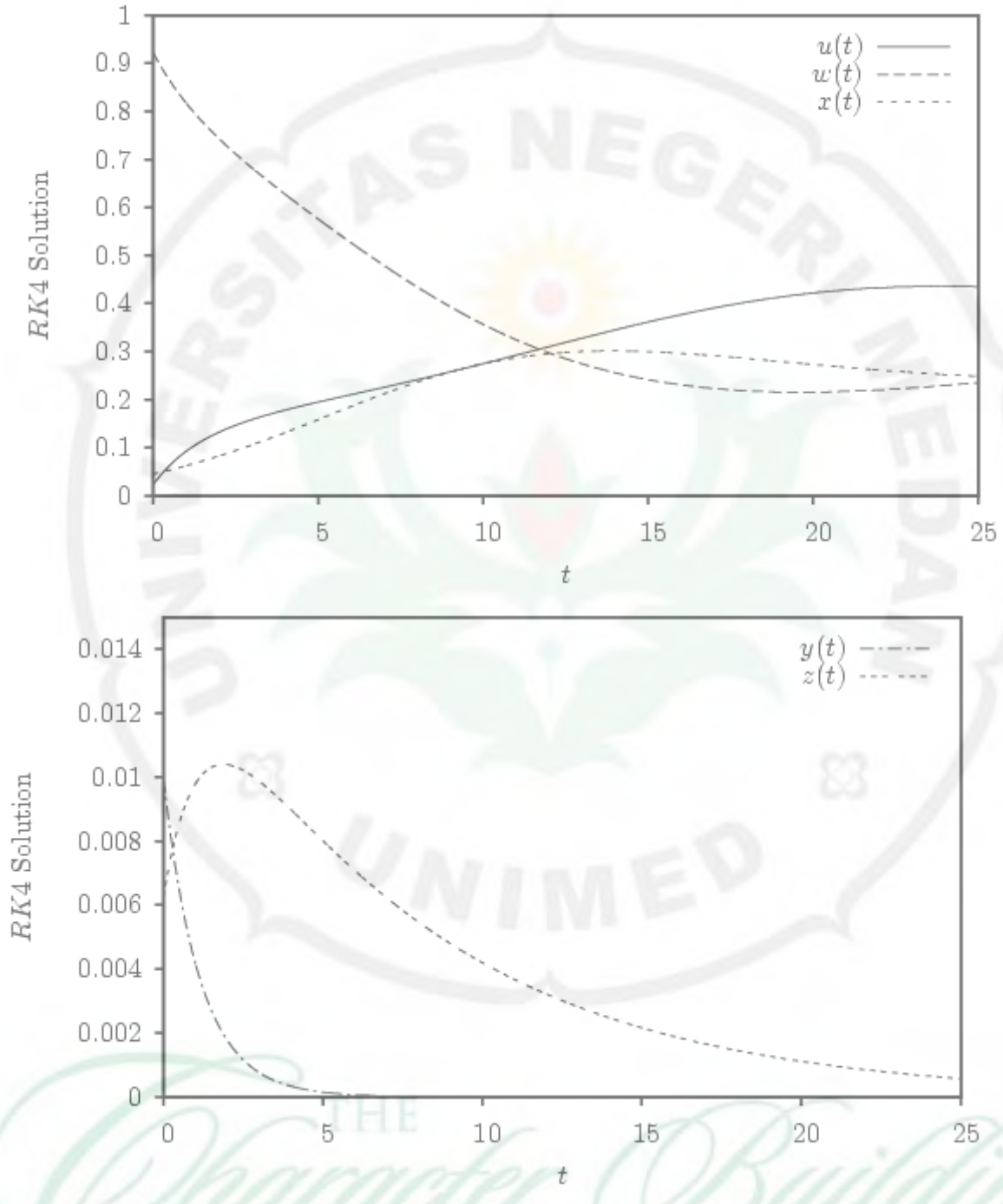
When $R_0 > 1$, it implies that endemic steady state is stable and the infection for a population, whereas when $R_0 \leq 1$ it implies that the uninfected steady state is stable. The other hand, the tuberculosis infects an individual, if $R_0 > 1$, otherwise.

A simulation carried out using MAPLE. Stability analysis tended to asymptotically stable. Illustration of the dynamics of each epidemic giving in Figure 1(a) and 1(b). Figure 1a shows the probability of vaccination,

susceptible, exposed, infected and recovery individuals that have $R_0 \leq 1$. It shows that North Sumatra is free disease area of TB. Otherwise, in Figure 1(b), $R_0 > 1$, it indicates North Sumatera is epidemic area of TB.



(a)



(b)

Figure 1. Transmission of Tuberculosis with: (a) dynamic system for $R_0 \leq 1$, (b) dynamic system for $R_0 > 1$

CONCLUSIONS

This paper has consider VSEIR model of Tuberculosis having infectious in latent, infected, vaccination and immune period. The breeding rate is derived. If $\boldsymbol{R_0 \leq 1}$ the free equilibrium is stable. So that the disease always dies out. Whereas, if $\boldsymbol{R_0 > 1}$, the disease free equilibrium become unstable in North Sumatera.

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