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Abstract Population problems can cause problems, both in terms of political, economic, socio-cultural life, defense and security, as well as other aspects of life concerning the use of natural resources and the environment. By knowing the population growth rate, the Medan city government can implement a policy to anticipate social problems that may arise. This study aims to determine the number and rate of female population growth in the city of Medan in 2025. The Leslie matrix model is a model that can be used to predict the number and rate of female population growth. The step taken to predict the number of population p in the following year is to form a column vector whose entry is the initial number of population for each age class. Next look for n (t + p) which is the total population for the following year using the formula $n(t+p) = A^p n(t)$ where A is the Leslie matrix. Furthermore, to predict the population growth rate using the Leslie matrix is to find the positive eigenvalues λ of the matrix A. Based on positive eigenvalues λ, three cases occur, namely: 1) the population will tend to increase if $\lambda > 1$; 2) the population will tend to decline if $\lambda < 1$; 3) the population will tend to be stable if $\lambda = 1$. Based on data analysis, the total female population in the city of Medan in 2025 is 1,576,294. Furthermore, the eigenvalues of $\lambda = 1{,}3673$ which mean that the number of female populations in the city of Medan tends to increase.

Keywords Leslie Matrix, Population Growth, Eigenvalues, Eigenvectors

1. Introduction

Medan City is the capital of North Sumatra Province. This city is the third-largest city in Indonesia after Jakarta and Surabaya, as well as the largest city outside Java Island with an area of 265.10 km^2. Based on data from the Central Statistics Agency (BPS) for Medan City in 2018 there is 2.264.145 soul, which consists of1.118.402 soul man and1.145.743 female soul. Compared to the total population in 2017, there was a population increase of16.720 souls (0,74percent) and the population density reached 8,541people / km^2. With the population density figure of the city of Medan, it may cause a lot of problems.

The population is an important asset of an area, but it should be noted that uncontrolled population growth will become a problem. The population problem in Indonesia has reached a disturbing level, both in terms of political, economic, socio-cultural life, defense and security, as well as other aspects of life about the use of natural resources and the environment, for example, uneven population density, large numbers of people. the number of unemployed and high population growth. Population problems in a narrow sense are always related to the number, structure, composition, and dynamic processes of the local population

With the problems described above, the government, especially the Medan City government, needs to be prepared and meet the needs of its citizens as a form of responsible government. Of course, the amount of effort the government has made is based on data and information, one of which is the rate of population and population growth in recent years. By knowing the population growth rate of Medan City in the past years, the city government can anticipate it in the long term by predicting the population for the next few years. Population predictions and population forecasts are often used as interchangeable terms. However, these two terms have very basic differences. Various literature states population projections as predictions (forecasts) which are based on certain rational assumptions built for future trends using statistical tools or mathematical calculations. On the other hand, population forecasting can be with or without assumptions or calculations. Without certain conditions or conditions or certain approaches.

By knowing the prediction of the number and rate of population growth, whether population growth is increasing, decreasing or remaining stable in the coming years, it will affect the development carried out which aims to provide clothing and food needs as basic needs, various educational, health and various other social facilities sufficient and evenly distributed in the context of increasing welfare. The population growth rate is the change in the number of people in a certain area each year.

Many models can be used to explain population growth. One of the models used by population experts is the Leslie model. Where this model uses a mathematical approach, namely the matrix. The Leslie matrix model is one of the models used by demographers, which was discovered by an ecologist named PH Leslie in 1945. The Leslie matrix has a unique shape, namely the Leslie matrix in the form of a square matrix with the first-row entry of the Leslie matrix consisting of fertility levels. female, the sub-diagonal contains the female's survival rate and the other entries are zero.

The Leslie Matrix Model can provide an overview of the dynamics of the process of growing a population in the long run, among others: regarding population growth, in the long run, the distribution of the population in age groups for the long term, and its application to a harvesting policy for a growing population [4]. The Leslie matrix is only used for one sex, and it is usually women who are considered. Because women reproduce, that means birth and birth will cause an increase in population

There are several studies on the Leslie matrix, among others, Yudha Pratama [7] conducted a study in East Java and the results showed several factors that influence population growth, namely fertility, survival, and age vulnerability of the population. Furthermore, Wahidah Sanusi, Sukarna and Nur Ridiawati [8] used the Leslie matrix to predict the number and rate of population growth in the city of Makassar in 2018. Their results showed that the number of female populations in the city of Makassar has always increased. Based on the description above and to avoid population problems, the Medan city government needs to know the prediction of the number and growth rate of Medan's population for the next few years.

2. Leslie's Matrix

One of the most common models of population growth used by population experts is the Leslie model. This model describes the growth of a human or animal population. The Leslie Matrix was discovered by an ecologist named P. H Leslie in 1945. The Leslie matrix can provide an overview of the dynamics of a population growth process, including population growth in the long run, distribution of population in age groups for the long term, and its application. on a harvesting policy for an established population.

Leslie's model using the following assumptions:

1. Only consider the number of female / female population.
2. The maximum age that can be reached by each individual is n year.
3. Age group of the population.
4. Endurance life *(Survival Rate)* for each age group to the next stage of age known.
5. Birth rate for each age group is known.
6. The distribution of initial ages is known

Define the age distribution $x^{(k)}$ at time with t_k

$$x^{(k)} = \begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ x_3^{(k)} \\ \vdots \\ x_n^{(k)} \end{bmatrix}$$

where $x_i^{(k)}$ is the number of girls in the age class at time. Thus, at the time, the number of girls in the first age class, only girls born between the time of and. The number of offspring produced for each age class can be calculated by multiplying the reproductive number for the age class by the number of girls in that age class for each group. The sum of these values gives the total number of offspring produced. Thus, it can be written mathematically as

$$x_1^{(k)} = L_1 x_1^{(k-1)} + L_2 x_2^{(k-1)} + \cdots + L_n x_n^{(k-1)} \qquad (1)$$

which states that the number of girls of age class 1 is equal to the number of girls born to girls aged class 1 between time and plus the number of girls born to girls aged class 2 between time and onwards plus the number of girls born to girls in age class - n between time t_k and t_{k-1}.

The number of women in the second age class group at the time of t_k were women in the first age class group at the time of t_{k-1} who were still alive at the time of t_k , or mathematically written as, $x_2^{(k)} = P_1 x_1^{(k-1)}$. Furthermore, the number of women in the 3rd age class group at the time of t_k were women in the first age class group at that time of t_{k-1} who were still alive at the time of t_k , or mathematically written as, $x_3^{(k)} = P_2 x_2^{(k-1)}$, in general, the number of women in the nth age class group at the time of t_k was women in the age class group $(n-1)$ at that time t_{k-1} who were still alive at that time of t_k , or mathematically written with, $x_n^{(k)} = P_{n-1}x_{n-1}^{(k-1)}$. The description above can be expressed in the form of a linear equation system, namely;

$$\begin{aligned} x_1^{(k)} &= L_1 x_1^{(k-1)} + L_2 x_2^{(k-1)} + \cdots + L_n x_n^{(k-1)} \\ x_2^{(k)} &= P_1 x_1^{(k-1)} \\ x_3^{(k)} &= P_2 x_2^{(k-1)} \\ x_n^{(k)} &= P_{n-1} x_{n-1}^{(k-1)} \end{aligned} \quad (2)$$

or in the form of a matrix to be

$$\begin{bmatrix} x_1^{(k)} \\ x_2^{(k)} \\ x_3^{(k)} \\ \cdot \\ \cdot \\ \cdot \\ x_n^{(k)} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{n-1} & a_n \\ b_1 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & b_{n-1} & 0 \end{bmatrix} \begin{bmatrix} x_1^{(k-1)} \\ x_2^{(k-1)} \\ x_3^{(k-1)} \\ \cdot \\ \cdot \\ \cdot \\ x_n^{(k-1)} \end{bmatrix}$$

or shorter is written as

$$x^{(k)} = Lx^{(k-1)} \quad (3)$$

Where

$x^{(k)} = \begin{bmatrix} x_1^{(k-1)} \\ x_2^{(k-1)} \\ x_3^{(k-1)} \\ \vdots \\ x_n^{(k-1)} \end{bmatrix}$, is the vector of the age distribution at time t_k

$x^{(k-1)} = \begin{bmatrix} x_1^{(k-1)} \\ x_2^{(k-1)} \\ x_3^{(k-1)} \\ \vdots \\ x_n^{(k-1)} \end{bmatrix}$, is the vector of the age distribution at time t_{k-1}

and

$$L = \begin{bmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \\ b_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & b_{n-1} & 0 \end{bmatrix} \quad (4)$$

called the Leslie matrix.

The birth process and the death process between two consecutive observations can be explained using the demographic parameters as shown in Table 1.

Table 1. Parameters in the Leslie Matrix

Model Parameters	Information
$a_i,\ i = 1,2,\cdots,n$	Average number of girls born to a woman during her age group -*i*
$b_i, i = 1,2,\cdots,n$	The number of women in the it age group that can be expected to be alive and up to the it age group

Based on the definition, it will be found that (i) $a_i \geq 0$ for $i = 1,2,\cdots,n$ and (ii) $0 < b_i \leq 1$ for $i = 1,2,\cdots,n$. It can be seen that it is not permissible to allow the existence of being equal to zero, because if this were to occur there would be no woman or female alive after age group i. It is also assumed that at least one is positive so that birth will occur. Each age group for which the value is positive is called the reproductive age group.

To determine the projection of population growth rate, further investigation of the eigenvalues of the Leslie matrix can be done. The eigenvalues of the Leslie matrix can indicate that the population growth rate tends to increase, decrease, or remain.

Theorem 2.1 The Leslie matrix has a single positive eigenvalue λ_1, this eigenvalue has multiplicity and is an eigenvector $\boldsymbol{x}_1$ all of the entries are positive.

Proof :

The eigenvalues of the Leslie matrix are the roots of the characteristic polynomial equation of the Leslie matrix. The characteristic equation of the matrix can be written as follows:

$$p(\lambda) = |\lambda I - L| = 0$$

$$\left| \lambda \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} - \begin{bmatrix} a_1 & a_2 & \cdots & a_{i-1} & a_i \\ b_1 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & b_{i-1} & 0 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} \lambda & 0 & 0 & \cdots & 0 \\ 0 & \lambda & 0 & \cdots & 0 \\ 0 & 0 & \lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda \end{bmatrix} - \begin{bmatrix} a_1 & a_2 & \cdots & a_{i-1} & a_i \\ b_1 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & b_{i-1} & 0 \end{bmatrix} \right| = 0$$

$$\lambda^n - a_1\lambda^{n-1} - a_2 b_1 \lambda^{n-2} - \cdots - a_i b_1 b_2 \cdots b_{i-1} = 0 \quad (5)$$

Equation (5) divided by λ^n , then we get:

$$\Leftrightarrow 1 - \frac{a_1}{\lambda} - \frac{a_2 b_1}{\lambda^2} - \frac{a_3 b_1 b_2}{\lambda^3} - \cdots - \frac{a_i b_1 b_2 \cdots b_{i-1}}{\lambda^n} = 0$$

$$\Leftrightarrow \frac{a_1}{\lambda}+\frac{a_2 b_1}{\lambda^2}+\frac{a_3 b_1 b_2}{\lambda^3}+\cdots+\frac{a_i b_1 b_2\cdots b_{i-1}}{\lambda^n}=1$$

Consider a polynomial equation

$$h(\lambda)= a_1\lambda^{-1}+a_2 b_1\lambda^{2}+a_3 b_1 b_2\lambda^{3}+\cdots+a_i b_1 b_2\cdots b_{i-1}\lambda^{-n}=0 \quad (6)$$

So that it is obtained

$$h(\lambda)=1 \quad (7)$$

It is known a_i, b_i to be positive. If the eigenvalues of the Leslie matrix λ_i are substituted for equation (6), let λ_i be positive for $0 \le i \le \infty$, then the values of $h(\lambda)$ will go towards zero and the monotone go down

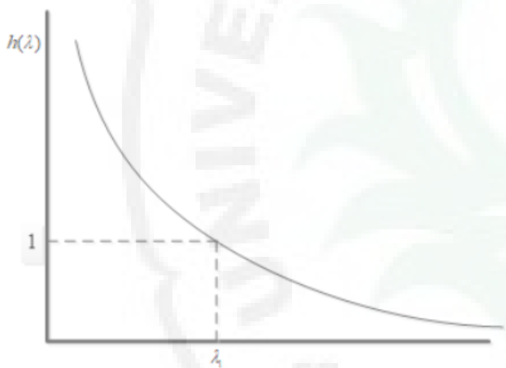

Figure 1. Graph of the $h(\lambda)$ function

From Figure 1, it is obtained that the value of each eigenvalue has exactly one solution value at $h(\lambda_1)$. So it can be concluded that the eigenvalues of the Leslie matrix differ from one another and there is a positive λ that is single, for example $\lambda=\lambda_1$ and $q(\lambda_1)=1$. Obtained also λ_1 has multiplicity equal to one.$h(\lambda_1)$

Given $\boldsymbol{x_1}$ is an eigenvector of A corresponding to that which satisfies λ_1

$$(\lambda_1 1 - A)\boldsymbol{x_1}=0.$$

For example

$$\boldsymbol{x_1}=\begin{bmatrix} x_1\\ x_2\\ x_3\\ \vdots\\ x_n\end{bmatrix}$$

So that

$$\left(\begin{bmatrix}\lambda_1 & 0 & 0 & \cdots & 0\\ 0 & \lambda_1 & 0 & \cdots & 0\\ 0 & 0 & \lambda_1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & \lambda_1\end{bmatrix} - \begin{bmatrix} a_1 & a_2 & \cdots & a_{i-1} & a_i\\ b_1 & 0 & \cdots & 0 & 0\\ 0 & b_2 & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & b_{i-1} & 0\end{bmatrix}\right)\boldsymbol{x_1}=\boldsymbol{0}$$

$$\begin{bmatrix}\lambda_1-a_1 & -a_2 & \cdots & -a_{i-1} & -a_i\\ -b_1 & \lambda_1 & \cdots & 0 & 0\\ 0 & -b_2 & \ddots & 0 & 0\\ \vdots & \vdots & \ddots & \ddots & \vdots\\ 0 & 0 & 0 & -b_{i-1} & \lambda_1\end{bmatrix}\boldsymbol{x_1}=\boldsymbol{0}$$

$$\begin{bmatrix}\lambda_1-a_1 & -a_2 & \cdots & -a_{i-1} & -a_i\\ -b_1 & \lambda_1 & \cdots & 0 & 0\\ 0 & -b_2 & \ddots & 0 & 0\\ \vdots & \vdots & \ddots & \ddots & \vdots\\ 0 & 0 & 0 & -b_{i-1} & \lambda_1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ \vdots\\ x_n\end{bmatrix}=\boldsymbol{0}$$

$$\begin{bmatrix}(\lambda_1-a_1)x_1-a_2x_2-\cdots-a_{i-1}x_{n-1}-a_ix_n\\ -b_1x_1+\lambda_1x_2\\ -b_2x_2+\lambda_1x_3\\ \vdots\\ -b_{i-1}x_{n-1}+\lambda_1x_n\end{bmatrix}=\begin{bmatrix}0\\0\\ \vdots\\0\\0\end{bmatrix}$$

then obtained

$$-b_1x_1+\lambda_1x_2=0 \quad \Leftrightarrow x_2=\frac{b_1x_1}{\lambda_1}$$

$$\Leftrightarrow x_2=\frac{b_1x_1}{\lambda_1} \quad (8)$$

$$-b_2x_2+\lambda_1x_3=0$$

$$\Leftrightarrow x_3=\frac{b_2x_2}{\lambda_1}\Leftrightarrow x_3=\frac{b_1b_2x_1}{{\lambda_1}^2} \quad (9)$$

$$-b_{i-2}x_{n-2}+\lambda_1x_{n-1}=0$$

$$\Leftrightarrow x_{n-1}=\frac{b_{i-2}x_{n-2}}{\lambda_1}$$

$$\Leftrightarrow x_{n-1}=\frac{b_1b_2x_1\dots b_{1-2}x_1}{{\lambda_1}^{n-2}} \quad (10)$$

$$-b_{i-1}x_{n-1}+\lambda_1x_n=0$$

$$\Leftrightarrow x_n=\frac{b_{i-1}x_{n-1}}{\lambda_1}\Leftrightarrow x_n=\frac{b_1b_2\cdots b_{i-1}x_1}{{\lambda_1}^{n-1}} \quad (11)$$

If equation (8), (9), (10), (11) is substituted for equation (7), then we get:

$$\lambda_1x_1-a_1x_1-a_2=\frac{b_1x_1}{\lambda_1}-a_3$$
$$=\frac{b_1b_2x_1}{{\lambda_1}^2}-\cdots-a_{n-1}\frac{b_1b_2\cdots b_{1-2}x_1}{{\lambda_1}^{n-2}}a_n\frac{b_1b_2\cdots b_{i-1}x_1}{{\lambda_1}^{n-1}}$$
$$=0$$

$$x_1\left(\lambda_1 - a_1 - a = \frac{b_1}{\lambda_1} - a_3 = \frac{b_2 x_2}{{\lambda_1}^2} - \cdots - a_{n-1}\frac{b_1 b_2 \cdots b_{i-2}}{{\lambda_1}^{n-2}} - a_n \frac{b_1 b_2 \cdots b_{i-1}}{{\lambda_1}^{n-1}}\right) = 0$$

then obtained

$$x_1 = 0 \quad (12)$$

It is known that $\boldsymbol{x_1}$ the nonzero vector corresponds to λ_1. From equation (12), if $\boldsymbol{x_1} = 0$, then the eigenvector corresponding to λ_1 is a zero vector. So much so that suppose $\boldsymbol{x_1} = t$, it is obtained

$$x_2 = \frac{b_1 t}{\lambda_1}$$

$$x_3 = \frac{b_1 b_2 t}{{\lambda_1}^2}$$

$$x_{n-1} = \frac{b_1 b_2 \ldots b_{1-2} t}{{\lambda_1}^{n-2}}$$

$$x_n = \frac{b_1 b_2 \ldots b_{1-1} t}{{\lambda_1}^{n-1}}$$

As a result, an eigenvector $\boldsymbol{x_1}$ that corresponds to shape λ_1 is obtained

$$\boldsymbol{x_1} = \begin{bmatrix} 1 \\ \frac{b_1}{\lambda_1} \\ \vdots \\ \frac{b_1 b_2 b_3 \ldots b_{1-2}}{{\lambda_1}^{n-2}} \\ \frac{b_1 b_2 b_3 \ldots b_{1-1}}{{\lambda_1}^{n-1}} \end{bmatrix} \quad (13)$$

with $t \in R - (0)$

Based on equation (13), it is found that the eigenvalue space $\boldsymbol{x_1}$ has one dimension, so it can be concluded that the geometric multiplicity is equal to one and it is obtained that the elements of the eigenvector $\boldsymbol{x_1}$ are positive numbers.

Theorem 2.2 If λ_1 is a single positive eigenvalue of a Leslie matrix and λ_1 is any real number eigenvalues or complex numbers of the Leslie matrix, then $|\lambda_k| \leq \lambda_1$

Proof :

Take any with $\lambda_k = re^{i\theta}e^{i\theta} = \cos\cos\theta + \sin\sin\theta$ so that $\lambda_k = r\ \cos\cos\theta + ir\ \sin\sin\theta$.

It will be shown that $r < \lambda_1$

From equation (7) it is known $q(\lambda) = 1$, so that it is obtained

$$h(\lambda_k) = h(\lambda_k) = 1$$

$$\Leftrightarrow \frac{a_1}{r\ \cos\cos\theta + ir\ \sin\sin\theta} + \frac{a_2 b_1}{(r\ \cos\cos\theta + ir\ \sin\sin\theta\ )^2} + \frac{a_3 b_1 b_2}{(r\ \cos\cos\theta + ir\ \sin\sin\theta\ )^3} + \cdots + \frac{a_3 b_1 b_2 \cdots b_{i-1}}{(r\ \cos\cos\theta + ir\ \sin\sin\theta\ )^n}$$

$$= \frac{a_1}{\lambda_1} + \frac{a_2 b_1}{{\lambda_1}^2} + \frac{a_3 b_1 b_2}{{\lambda_1}^3} + \cdots + \frac{a_3 b_1 b_2 \cdots b_{i-1}}{(\lambda_1)^n} = 1$$

$$\Leftrightarrow \frac{a_1}{\lambda_1} = \frac{a_1}{r\ \cos\cos\theta + ir\ \sin\sin\theta}$$

by taking the real part of the two equations we get:

$$\frac{a_1}{r\cos\cos\theta} = \frac{a_1}{\lambda_1}$$

so that $r = \frac{\lambda_1}{\cos\cos\theta}$ and it can be concluded that

$$r \leq \lambda_1 \text{ and } |\lambda_k| \leq \lambda_1.$$

It is known that $r \leq \lambda_1$ it is evident that for anything λ_k that is a real or complex number is valid $|\lambda_k| \leq \lambda_1$.

Definition 2.1 Given $\lambda_1, \lambda_2, \ldots, \lambda_n$ is the eigenvalue of matrix A of size $n\,x\,n$, λ_1 is said to be the dominant eigenvalue of A if $|\lambda_k| > |\lambda_i|$ with $i = 1,2,\cdots,n$.

Suppose that the Leslie A matrix can be diagonalized, then there is an eigenvalue, for example $\lambda_1, \lambda_2, \lambda_2, \ldots, \lambda_n$, and it has an eigenvector $x_1, x_2, x_3, \ldots, x_n$. Suppose that a matrix is formed

$$P = [x_1|x_2|x_3|\ldots|x_n].$$

then there is P^{-1} which is the inverse matrix of P. The diagonalization of the matrix A takes the form:

$$A = P\begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix} P^{-1}$$

For A^p with $p = 1,2,\ldots,n$, the diagonalization equation becomes

$$A^p = P\begin{bmatrix} \lambda_1^p & 0 & \cdots & 0 \\ 0 & \lambda_2^p & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n^p \end{bmatrix} P^{-1}$$

Suppose there is n (t) which is the vector of the initial distribution of a population, if multiplied by the diagonalization equation, then the equation becomes

$$,A^p n(t) = P\begin{bmatrix} \lambda_1^p & 0 & \cdots & 0 \\ 0 & \lambda_2^p & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n^p \end{bmatrix} P^{-1} n(t)$$

For example, if the entry results in a multiplication

$$P^{-1}n(t)=\begin{bmatrix}c_1\\c_2\\\vdots\\c_n\end{bmatrix}, \qquad (14)$$

then

$$n(t+p)=[x_1|x_2|x_3|\dots|x_n]\begin{bmatrix}\lambda_1^p & 0 & \cdots & 0\\ 0 & \lambda_2^p & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda_n^p\end{bmatrix}\begin{bmatrix}c_1\\c_2\\\vdots\\c_n\end{bmatrix}$$

$$\Leftrightarrow n(t+p)=[x_1|x_2|x_3|\dots|x_n]\begin{bmatrix}\lambda_1^p c_1\\\lambda_2^p c_2\\\vdots\\\lambda_n^p c_n\end{bmatrix}$$

$$\Leftrightarrow n(t+p)=x_1\lambda_1{}^P c_1+x_2\lambda_2{}^P c_2+\dots+x_n\lambda_n{}^P c_n$$

It will be shown that if λ_1 it is the dominant eigenvalue it will affect population growth. The two sides are divided by $\lambda_1{}^P$, then the equation becomes

$$\frac{1}{\lambda_1{}^P}n(t+p)=P\begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & \left(\frac{\lambda_2}{\lambda_1}\right)^p & \cdots & 0\\ & & \ddots & \vdots\\ \vdots & \vdots & & \\ 0 & 0 & \cdots & \left(\frac{\lambda_n}{\lambda_1}\right)^p\end{bmatrix}P^{-1}n(t)$$

It is known that λ_1 the dominant eigenvalues of the Leslie matrix, then $\left(\frac{\lambda_n}{\lambda_1}\right)$, for $n=1,2,\dots,n,$ and are obtained $\left(\frac{\lambda_n}{\lambda_1}\right)^p\to 0$ when $p\to\infty$.

A limit is formed

$$lim_{p\to\infty}\frac{1}{\lambda_1{}^P}n(t+p)$$

$$=P\begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0\end{bmatrix}P^{-1}n(t) \qquad (15)$$

Given that P^{-1} is a matrix with the order nxn, and $n(t)$ the vector of the initial distribution with the order $nx1$. If P^{-1} and $n(t)$ are multiplied according to the multiplication properties of the matrix, then the matrix result will be ordered $nx1$. Both the P^{-1} and $n(t)$ matrices have entries which are constants. Substituting equation (14) to equation (15), is obtained

$$\Leftrightarrow lim_{p\to\infty}\frac{1}{\lambda_1{}^P}n(t+p)=P\begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0\end{bmatrix}\begin{bmatrix}c_1\\c_2\\\vdots\\c_n\end{bmatrix}$$

$$\Leftrightarrow lim_{p\to\infty}\frac{1}{\lambda_1{}^P}n(t+p)$$

$$=[x_1|x_2|x_3|\dots|x_n]\begin{bmatrix}1 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & 0\end{bmatrix}\begin{bmatrix}c_1\\c_2\\\vdots\\c_n\end{bmatrix}$$

$$\Leftrightarrow lim_{p\to\infty}\frac{1}{\lambda_1{}^P}n(t+p)=[x_1|x_2|x_3|\dots|x_n]\begin{bmatrix}c_1\\0\\\vdots\\0\end{bmatrix}$$

$$\Leftrightarrow lim_{p\to\infty}\frac{1}{\lambda_1{}^P}n(t+p)=c_1x_1+0x_2+0x_3\dots+0x_n$$

$$\Leftrightarrow lim_{p\to\infty}\frac{1}{\lambda_1{}^P}n(t+p)=c_1x_1 \qquad (16)$$

From equation (16) an approximation is obtained

$$n(t+p)=c_1\lambda_1{}^P x_1$$

$$\Leftrightarrow n(t+p)=c_1\lambda_1{}^{P-1}x_1$$

such that $n(t+p)$

$$=\lambda_1{}^P c_1x_1 \ \Leftrightarrow n(t+p)=c_1{}^1\lambda_1{}^{p-1}c_1x_1$$

obtained $n(t+p)=\lambda_1 n(t+(p-1))$ (17)

From equation (17), it is found that, if for any value p that determines the next year in the population, and it is known that $\lambda_1=1$ the dominant eigenvalue of the Leslie matrix, it can be concluded that the next age distribution vector will always be the same as the previous age vector. So as to result:

1. If $\lambda_1<1$ is known, the population of all age classes tends to decline.
2. If $\lambda_1=1$ is known, the population in all age classes tends to remain.
3. If $\lambda_1>1$ is known, the population of all age classes tends to increase.

3. Research Result

In Table 2. below, data obtained from the BPS of Medan city is provided, namely the number of female residents in 2010 and 2015 in the city of Medan, grouped by age.

Data obtained from BPS are data on female population by age, data on the birth of girls according to maternal age at delivery, and data on female population mortality. The data is arranged in the following frequency distribution table.

Table 2. Female Population in Medan City 2010 and 2015

Age Class	Number of Women in 2010	Number of Women in 2015
0-4	92,857	99,065
5-9	93,532	95,441
10-14	91,828	89,405
15-19	107,423	109,850
20-24	123,092	128,830
25-29	103,459	100,090
30-34	87,265	90,398
35-39	80,759	84,551
40-44	71,727	75,953
45-49	59,997	65,817
50-54	49,244	56,676
55-59	34,282	45,175
60-64	22,555	31,355
65-69	17,556	19,903
70-74	12,384	13,714
75+	12,688	12,364
Total	1,060,648	1,118,587

From Table 3, the initial population of women in Medan is 1,060,648 people, the number of female births (A_i) is 270,612 and the number of female population deaths (Bi) is 6,597.

Furthermore, the Leslie matrix model is formed, therefore the element of female fertility level is needed, for example (a_i) and the element of female survival rate (b_i). The formula for finding ai and bi is as follows.

$$a_i = \frac{A_i}{x_i^0} \text{ and } b_i = 1 - \frac{B_i}{x_i^0}$$

by:

A_i =Number of births in age class to $-i$

B_i =Number of female deaths in age class to$-i$

X_i^0 = The number of initial population of women in age class to $-i$

Table 3. Female Population in Medan, 2010-2015

Age Class	Age Interval	Number of Women in 2010 (x_i^0)	Birth (A_i)	Dead (B_i)
1	0-4	92,857	0	527
2	5-9	93,532	0	483
3	10-14	91,828	0	638
4	15-19	107,423	21,188	712
5	20-24	123,092	62,544	487
6	25-29	103,459	80,023	512
7	30-34	87,265	48,995	338
8	35-39	80,759	40,171	498
9	40-44	71,727	17,691	446
10	45-49	59,997	0	310
11	50-54	49,244	0	272
12	55-59	34,282	0	304
13	60-64	22,555	0	385
14	65-69	17,556	0	367
15	70-74	12,384	0	171
16	75+	12,688	0	147
total		1,060,648	270,612	6,597

Table 4. Levels of Fertility and Survival Women in Medan in 2010 - 2015

Age Class	Fertility Rate (a_i)	Level of Resistance (b_i)
1	0	0.9944
2	0	0.9949
3	0	0.9931
4	0.196	0.9934
5	0.508	0.9961
6	0.773	0.9951
7	0.561	0.9962
8	0.497	0.9939
9	0.246	0.9938
10	0	0.9949
11	0	0.9945
12	0	0.9912
13	0	0.983
14	0	0.9791
15	0	0.9862
16	0	0.9885

From Table 4, above, the Leslie matrix model is obtained. The resulting Leslie matrix is a matrix with the order of 16 × 16 whose elements consist of the female fertility rate (a_i) in the first row and the female survival rate (b_i) in the diagonal row of the female population.

$$L=\begin{bmatrix}
0 & 0 & 0 & 0196 & 0508 & 0773 & 0561 & 0497 & 0246 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
09944 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 09949 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 09931 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 09934 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 09961 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 09951 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 09962 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 09939 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 09938 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 09949 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 09945 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 09912 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0983 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 09791 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 09862 & 0
\end{bmatrix}$$

then the predicted number of female population in 2025 is:

$$x^{(2)}=\begin{bmatrix}
0 & 0 & 0.1946 & 0.5046 & 0.7700 & 0.5583 & 0.4951 & 0.2445 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.1949 & 0.5052 & 0.7687 & 0.5579 & 0.4942 & 0.2446 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.9893 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.9880 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.9865 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.9895 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.9912 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.9913 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.9901 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9877 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9887 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9894 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9857 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9743 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9625 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9656 & 0 & 0
\end{bmatrix} \cdot \begin{bmatrix} 92.857 \\ 93.532 \\ 91.828 \\ 107.423 \\ 123.092 \\ 103.459 \\ 87.265 \\ 80.795 \\ 71.727 \\ 59.997 \\ 49.244 \\ 34.282 \\ 22.555 \\ 17.556 \\ 12.384 \\ 12.688 \end{bmatrix} = \begin{bmatrix} 287.580 \\ 268.801 \\ 91.866 \\ 92.413 \\ 90.593 \\ 106.298 \\ 122.011 \\ 102.561 \\ 86.403 \\ 79.804 \\ 70.919 \\ 59.363. \\ 48.542 \\ 30.480 \\ 21.708 \\ 16.952 \end{bmatrix}$$

$$x^{(2)} = 287.580 + 268.801 + 91.866 + 92.413 + 90.593 + 106.298 + 122.011 + 102.561 + 86.403 + 79.804 + 70.919 + 59.363 + 48.542 + 30.480 + 21.708 + 16.952 = 1.576.294$$

To find out the prediction of the growth rate of the female population in the city of Medan, a search for eigenvalues was carried out. Because the L matrix is quite large, the authors looked for the eigenvalues of the Leslie matrix with the help of matlab software and obtained:

$\lambda_1 = 1.3673$, $\lambda_2 = 0.4724 + 0.1704i$, $\lambda_3 0.7424 - 0.1704i$, $\lambda_4 = -0.3623 + 0.9047i$, $\lambda_5 = -0.3623 - 0.9047\text{i}$, $\lambda_6 = -0.2055 + 0.6619\text{i}$, $\lambda_7 = -0.2055 - 0.6619\text{i}$, $\lambda_8 = 0.5883 + 0.0829\text{i}$, $\lambda_9 = -0.5883 - 0.0829\text{i}$, $\lambda_{10} = 0$, $\lambda_{11} = 0$, $\lambda_{12} = 0$, $\lambda_{13} = 0$, $\lambda_{14} = 0$, $\lambda_{15} = 0$, $\lambda_{16} = 0$ and dominan eigenvalue is $\lambda_1 = 1.3673$. So it can be concluded that the female population growth in the city of Medan will ten to increase.

4. Conclusions

Based on the results and discussion, the following conclusions were obtained:

a). In the prediction analysis the population size obtained $x^{(2)} = 1{,}576{,}294$. So it can be seen that the number of female population in 2025 has increased.

b). The results of the application of the Leslie matrix obtained the eigen value or $\lambda_1 > 1$. This indicates that the number of female population in Medan City in 2025 will increase.

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