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Solving Quadratic Assignment Problem Using Forward and Backward Exchange Algorithm

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Abstract

Quadratic assignment problem is one of the combinatorial optimization problems of deciding the placement of facilities in specified locations in such a way as to minimize a nonconvex objective function expressed in terms of flow between facilities, and distance between location. Since QAP is NP-hard problem, and even finding an ε-approximate solution is a difficult problem, therefore to get a 'good' starting point is necessary, in order to obtain a better optimal solution. In this paper we propose a random point strategy to get initial starting point and then use forward exchange algorithm and backward exchange algorithm to get 'optimal' solution. As a computational experience we solved the problem of Had12, Esc 16b, Esc 16c and Esc 16h from QAPLIB. Finally, we present a comparative study between our proposed algorithm and Data –Guided Lexisearch Algorithm. The computational study shows the effectiveness of our proposed algorithm.

Keywords: Backward Exchange Algorithm, Forward Exchange Algorithm, Quadratic Assigment Problem, Random Point Strategy,

INTRODUCTION

Quadratic assignment problem (QAP) is in simplest form concerned with locating facilities on locations, such that total transportation costs are minimized. The transportation cost incurred by locating two facilities is proportional both to the flow of transportation between the facilities and the distance between their locations. In 1957 QAP was introduced by Koopmann and Beckmann as a mathematical model related to economic activities. The QAP however is not only nonlinear but also not unimodal, so many researches attracted to investigate this problem. An heuristic yielding a good solution seems a more appropriate course of action than attempting an

optimal solution, and many of the papers aim at achieving improvements to an initial assignment.
The QAP has been applied to many fields such as backboard wiring, typewriter keyboards and control panel design, scheduling, numerical analysis, storage-and-retrieval, and many others. One of the oldest applications of the QAP is the assignment of specialized rooms in a building (Elshafei, 1977). In this case, a_{ij} is the flow of people that must go from service i to service j and b_{ij} is the time for going from room i to room j. The other one is application in turbine runner balancing (Laporte and Mercure, 1988). A recent application is the assignment of gates to airplanes in an airport (A. Bouras *et. al.*, 2014); in this case, a_{ij} is the number of passengers going from airplane i to airplane j (a special "airplane" is the main entrance of the airport) and b_{kl} is the walking distance between gates k and l.

LITERATURE REVIEW

The QAP was one of the first problems solved by metaheuristics methods first conceived in the 1980's. Burkard and Rendl (1984) proposed a simulated annealing procedure that was able to find much better solutions than all the previously designed heuristic methods. Six years later, Connolly (1990) proposed an improved annealing scheme. His method is easy to set up, since the user has to select the number of iterations; and all other parameters are automatically computed. At around the same time, Skorin-Kapov (1990) proposed a tabu search. Then, Taillard (1991) proposed a more robust tabu search, with fewer parameters and running *n* times faster than the previous implementation. Even though Taillard's method was proposed over 24 years ago, it remains one of the most efficient for some problem instances. In 1992 Hadley *et. al* propose a new lower bound (Hadley *et. al*,1992), two years later Battiti and Tecchiolli proposed the reactive tabu search (Battiti and Tecchiolli, 1994a), also the comparison of tabu search techniques and simulated annealing (Battiti and Tecchiolli, 1994b), then the star-shape diversification approach of Sondergeld and Voss (1996). Recent T. James *et. al.* (2007), addressed a multi-start tabu search and diversification strategies for the QAP.
Genetic algorithms have also been proposed, for instance by Tate and Smith (1995), but hybrid approaches, such as those of Fleurent and Ferland (1994), Ahuja *et al.* (2000), Drezner (2002a, 2002b), are more efficient. Another heuristic, GRASP (greedy randomized adaptive search procedure) was proposed by Li, Pardalos and Resend (1994). An ant colony system approaches by Taillard (1998) , Gambardella *et al.*, (1999), Stutzle and Hoos 1999,) have been proposed, as well as a scatter search (Cung *et al.*, 1997). Some of these methods have been compared in Taillard (1995) who showed that the efficiency of these methods strongly depends on the problem instance type to which they are applied.
In 2005 Ahyaningsih *et. al.* proposed some new results and generalizations for QAP (2005), then in 2006 Ahyaningsih proposed improved strategy for solving QAP (2006). Recently, Ahmed ZH., (2011a) proposed a data-guided lexisearch algorithm

(DGLSA) for the asymmetric traveling salesman problem. Still in this year, Ahmed ZH., (2011b) solved the bottleneck travelling salesman problem using DGLSA. In 2014 Ahmed ZH solved QAP using DGLSA, (2014). One year later Ahyaningsih and Sitompul proposed a combined strategy for solving QAP (2015).

FORMULATION OF QUADRATIC ASSIGNMENT PROBLEM

QAP is combinatorial problem of deciding the placement of facilities in specified locations in such a way as to minimise a quadratic objective function. Consider the problem of locating *n* facilities in *n* given locations. If the flow f_{ik} between each pair of facility *i* and facility *k* and the unit transportation cost (or distance) d_{jl} between locations *j* and *l* are known, then the problem is defined to be

minimize $$\Phi = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$

subject to
$$\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1,\dots,n$$
$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1,\dots,n$$
$$0 \le x_{ij} \le 1$$
$$x_{ij} \quad \text{integer}$$

Matrices [f_{ik}] and [d_{jl}] are assumed to be symmetric. The assignment variable x_{ij} has a value 1 if facility *i* is at location *j*, and is zero otherwise. The constraints reflect the fact that each location can be assigned to only one facility, and each facility can be assigned to only one location.

Generally the QAP is a non-convex problem so any solution obtained will necessarily be a local optimum and not a global optimum.

ALGORITHM FOR SOLVING QAP

Since the quadratic form may be non-convex, the chances of obtaining a good integer feasible solution are considerably enhanced by paying some attention to the starting point for the search procedure.

In this case we create a computer program that would generate a random assignment. Then we calculate the value of objective function using this random assignment. In order to get a good initial starting point it is neccessary to input an arbitrary value of objective function θ enough big say θ=10000000, so its value always greater than $\bar{\theta}$, at last from a numbers of iteration $\theta\,min$ will be reached, match to the assigment x_i. Input x_i to the objective function θ in forward exchange algorithm and backward exchange algorithm. This process repeated according to the number of iteration. The last θmin will be reached.

The steps used to get a random initial starting point are written as follows :

1. Read θ= 10000000, matrix [f_{ik}] dan [d_{jl}], n = problem dimension and *ITEM* as the number of iteration, a = 1
2. Generate random permutation, x_i= randperm (n); for every facility i, i = 1, 2, …, n.
3. $x_{ij}=1$ for gen $x_i=j$; $x_{ij}= 0$ for generate $x_i \neq j$; i,j = 1,2,..,n
4. Calculate the objective function value

$$\bar{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$

5. if $\bar{\theta} < \theta$, then $\theta = \bar{\theta}\,; a = a+1$
6. if $a < ITEM$, back to (2)
7. STOP

Forward Exchange Algorithm:

1. Input n as problem dimension.
2. Read flow matrix [f_{ik}], distance matrix [d_{je}], initial asssignment [x_{ij}]
3. Calculate $\theta = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$
4. Generate $x_i=j$ for $x_{ij}=1$; $i,j=1, \ldots, n$
5. a=1
6. b=1
7. c=Gen $x_{i=a}$; Gen $x_{i=a}$=Gen $x_{i=a+b}$; Gen $x_{i=a+b}=c$
8. x_{ij}=1 for Gen $x_i=j$, $x_{ij}=0$ for Gen $x_i \neq j$, $i,j=1,\ldots, n$
9. Calculate objective function

$$\bar{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$

10. if $\bar{\theta} < \theta$ then $\theta = \bar{\theta}$ go to (14)

11. Gen $x_{i=a+b}$ = Gen$x_{i=a}$; Gen $x_{\mathrm{i=a}}$=c
12. $b = b+1$
13. if $b \leq n - a$ go to (7)
14. $a = a$+1
15. if $a \leq n$ go to (6)
16. STOP

Backward Exchange Algorithm :

1. Input n as problem dimension
2. Read flow matrix [f_{ik}], distance matrix [$d_{je,}$], initial assignment X = [x_{ij}.]
3. Calculate $\theta = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$
4. Gen x_{i}=j, for x_{ij}=1 ; i,j = 1, …, n
5. $a = n$
6. $b = n$
7. c=Gen $x_{i=a}$; Gen $x_{i=a}$= Gen $x_{i=b}$; Gen $x_{i=b}$ =c
8. x_{ij}=1 for Gen x_i=j, x_{ij}=0 for Gen x_i≠j, i,j=1,…, n
9. Calculate objective function

   $$\overline{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$
10. if $\overline{\theta} < \theta$ then $\theta = \overline{\theta}$ go to (14)
11. Gen $x_{i=b}$ = Gen $x_{i=a}$; Gen $x_{\mathrm{i=a}}$=c
12. $b = b$-1
13. if $b \geq a + 1$ go to (7)
14. $a = a$-1
15. if a > 1 go to (6)
16. STOP

COMPUTATIONAL EXPERIENCE

To run the program we use laptop with processor Intel (R) core (TM) i3-3217U, CPU 1.80 GHz RAM 4.00 GB.

12 × 12 Problem :

The 12 × 12 QAP is adopted from Hadley (1992). This is a large scale problem with 144 binary variables. To solve the problem, we input n as dimention of problems, number of iteration, matrix flow [f_{ik}] and matrix distance [$d_{je,}$]. In each iteration we use random point strategy to get the initial assignment of the QAP and then we use forward exchange strategy and backward exchange strategy to get the integer feasible solution.

We try many kind of iterations.

The result can be seen in table 1 below.

Table 1. The Search Table for Had12

Number of Iteration	Objective Value	Permutation	Running Time (Second)
500	1660	3 10 11 2 12 7 5 1 8 6 4 9	57.997902
600	1664	3 10 2 12 11 5 6 7 8 1 4 9	70.160402
700	1660	3 10 12 2 11 5 6 7 8 1 4 9	81.361419
750	1654	3 10 11 2 12 5 7 6 8 1 9 4	86.357992
800	1654	8 10 2 11 12 5 6 7 3 1 4 9	92.285597
850	1654	8 10 2 11 12 5 7 6 3 1 4 9	99.568710
900	1656	3 10 5 2 12 11 7 1 8 6 4 9	104.039284
950	1656	3 10 5 2 12 11 7 1 8 6 4 9	110.342666
1000	1660	3 10 11 2 12 6 5 1 8 7 4 9	117.018335
1050	1652(OPT)	3 10 11 2 12 5 6 7 8 1 4 9	122.512347

From table 1 we can conclude :

1. By the increasing of number iteration, the objective value approach to the optimum value.
2. On iteration 1050 the optimum value was reached, as reported on QAPLIB.
3. There is a number of iteration have same objective value but different permutation (just in case, based on the matrix flow [f_{ik}] and matrix distance [$d_{je,}$])

16 x 16 Problem :

The 16 × 16 QAP is adopted from Escermann (1990). This is a large scale problem with 256 binary variables. We try the number of iteration is 500, no other number of iteration, because by using this number of iteration we found the optimal solution according to the QAPLIB, Burkard, *et. al.* (1997).

The result can be seen in table 2 below.

Table 2. The Search Table for Esc16

Instance	Number of Iteration	Objective Value	Permutation	Running Time (Second)
Esc 16b	500	292 (OPT)	2 3 16 9 7 13 1 5 15 11 14 4 12 8 10 6	371.654879
Esc 16c	500	160 (OPT)	5 16 15 8 7 6 11 3 4 9 2 10 12 14 1 13	270.088915
Esc 16h	500	996 (OPT)	16 15 8 5 14 9 10 6 1 2 13 4 3 11 12 7	326.956387

If we compare our proposed algorithm and Data–Guided Lexisearch Algorithm (DGLSA) by Ahmed (2014), the result showed in table 3 :

Table 3. The Comparison of DGLSA and Our Proposed Algorithm

Instance	BKV	DGLSA			Our Proposed Algorithm		
		BSV	Error (%)	Running Time (Second)	BSV	Error (%)	Running Time (Second)
Esc 16b	292	292	0.00	14400.00	292	0.00	371.654879
Esc 16c	160	160	0.00	2984.60	160	0.00	270.088915
Esc 16h	996	996	0.00	1298.50	996	0.00	326.956387

BKV = Best known value (from QAPLIB)
BSV = Best solution value
Error = { (BSV-BKV)/BKV } x 100%

From table 3 we show that our proposed methods much more better than Data–Guided Lexisearch Algorithm (DGLSA).

CONCLUSION

For size problem 12 optimum value reached at iteration 1050, meanwhile for size problem 16 optimum value reached at iteration 500 (it caused matrix Esc16 more simple then matrix Had12 (as can be seen from QAPLIB), so the problem not only based on the size but also on the matrix itself).

Generally, the greater problem instance the greater number of iteration, but in this case not like that.

Overall the procedure described here has proved successful in obtaining integer feasible solution to the Quadratic Assignment Problem in relatively short computing times (as can be seen from the running time).

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