Developing A Combined Strategy For Solving Quadratic Assignment Problem

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Abstract: The quadratic assigment problem (QAP) is one of the most interesting and most challenging combinatorial optimization problems in existence. In this paper we propose a random point strategy to get a starting point, and then we use a combination methods to get ‗optimal' solution. As a computational experience we've solved QAP 30 x 30 adopted from Nugent and backboard wiring problem 42 × 42, adopted from Skorin-Kapov.

Index Terms: Combination Methods, Combinatorial Optimization Problem, Quadratic Assigment Problem, Random Point Strategy.

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1 INTRODUCTION

The Quadratic assignment problem (QAP) is in simplest form concerned with locating facilities on locations, such that total transportation costs are minimized. The transportation cost incurred by locating two facilities is proportional both to the flow of transportation between the facilities and the distance between their locations. In 1957 QAP was introduced by Koopmann and Beckmann as a mathematical model for the location of a set of indivisible economic activities. The QAP however is not only nonlinear but it is not unimodal. As a consequence this problem class has attracted active investigation by numerous researches. An heuristic yielding a good solution seems a more appropriate course of action than attempting an optimal solution, and many of the papers aim at achieving improvements to an initial assignment. Due to its nature the QAP can be regarded as a combinatorial optimization problem and it belongs to one of the most difficult problem to be solved. In general, the QAP with order n > 30 can not be solved in reasonable time. Due to its complexity the QAP is NP-hard optimization problem, Sahni and Gonzales (1976), therefore practically the most recommended approach for solving the QAP is to apply heuristic algorithm. The objective function in the QAP is in quadratic form and it is a non convex function. In this paper we propose a combination of exact and heuristic methods for solving the QAP. The first heuristic is to find the initial starting point. This strategy is necessary due to the non convexity of the function, then we solve the quadratic programming problem using this starting point. At the end we use another heuristic to get the result of the QAP.

2 SURVEY LITERATURE

The QAP was first proposed by Koopmans and Beckmann (1957) as a mathematical model related to facility - location activities. Since then, there are diverse real world applications. One of the famous application is proposed by Steinberg (1961). He used the QAP to minimize the total amount of connections between components in a backboard wiring, known as backboard wiring problem. Dickey and Hopkins (1972) applied the QAP for the assignment of buildings in a University campus, Elshafei (1977) applied the QAP for the hospital planning. Recent applications can be found in Bisaillon, S., *et al*. (2011), he applied the QAP for the aircraft and passenger recovery problem, Ribeiro, G. M., *et al*,(2011) applied the QAP for map labeling, Chiarandini, M., *et al*. (2012) applied the QAP for the balanced academic curriculum problem revisited. Many papers discuss about how to solve the QAP. In 1978 Burkard and Stratmann try to solve the QAP with numerical investigation. Then an improved annealing scheme are suggested in Connolly (1990). In the same year Skorin-Kapov (1990) proposed tabu search. One year later, Taillard (1991) proposed a more robust tabu search. His methods running faster and fewer parameters than the previous methods. Furthermore, Battiti and Tecchiolli (1994a) proposed a reactive tabu search, and still in the same year Battiti and Tecchiolli (1994b) made a comparison between tabu search techniques and simulated annealing. Recent T. James *et al* (2007) addressed a multi-start tabu search and diversification strategies for the quadratic assignment problem. The other methods is the star-shaped diversification approach proposed by Sondergeld and S. Vob (1996), then in the 2000s, improved tabu search algorithm was proposed by Misevicius (2003,2005). GRASP (Greedy randomized adaptive search procedure) was proposed by Li, Pardalos and Resend (1994), genetic algorithms has been proposed by Tate and Smith (1995), scatter search proposed by Cung et al. (1997), An ant colony system proposed by Gambardella *et al*., (1999), Genetic hybrid approaches was proposed by Fleurent and Ferland (1994), Descent genetic algorithm proposed by Ahuja *et al.* (2000), and New genetic algorithm and Robust heuristic algorithm proposed by Drezner (2002a, 2002b). A relatively new research conducted by Drezner (2006), who argued about the cluster point and gray pattern, Ahyaningsih F. (2006) solved QAP by heuristic kelayakan Methods, F. Rendl and R. Sotirov (2007) argued about the limit using the bundle method, Papamanthou C. et. al. (2008) argued about the case that one of the exterior point algorithm, Misevicius and Rubliauskas (2009) proposed a hybrid genetic algorithm to solve QAP structured, G.G. Zabudskii and A. Yu. Lagzdin (2010) solved the QAP using a polynomial algorithm, and Ahmed ZH (2014) solved the QAP with Lexisearch -Guided Data Algorithm.

3 THE QUADRATIC ASSIGNMENT PROBLEM

This is combinatorial problem of deciding the placement of facilities in specified locations in such a way as to minimise a quadratic objective function. Consider the problem of locating *n* facilities in *n* given locations. If the flow f_{ik} between each pair

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of facility *i* and facility *k* and the unit transportation cost (or distance) d_{jl} between locations *j* and *l* are known, then the problem is defined to be

$$\text{minimize} \quad \Phi = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = 1 \quad i = 1,\ldots,n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1,\ldots,n$$

$$0 \le x_{ij} \le 1$$

$$x_{ij} \quad \text{integer}$$

Matrices [f_{ik}] and [d_{jl}] are assumed to be symmetric. The assignment variable x_{ij} has a value 1 if facility *i* is at location *j*, and is zero otherwise. The constraints reflect the fact that each location can be assigned to only one facility, and each facility can be assigned to only one location. Generally the QAP is a non-convex problem so any solution obtained will necessarily be a local optimum and not a global optimum.

4 STRATEGY TO GET INITIAL STARTING POINT

Since the quadratic form may be non-convex, the chances of obtaining a good integer feasible solution are considerably enhanced by paying some attention to the starting point for the search procedure. We use random point strategy to get an initial assignment. In this case we create a computer program that would generate a random assignment. Then we calculate the value of objective function using this random assignment. In order to get a good initial starting point it is neccessary to input an arbitrary value of objective function θ. The steps used to get a random initial starting point are written as follows :

1. Read θ = 10000000, matrix [f_{ik}] dan [d_{jl}], *n* = problem dimension and *ITEM* as the number of iteration, *a* = 1
2. Generate random permutation, x_i= randperm (*n*); for every facility *i*, *i* = 1, 2, …, *n*.
3. $x_{ij}=1$ for gen $x_i=j$; $x_{ij}= 0$ for generate $x_i \neq j$; i,j = 1,2,..,n
4. Calculate the objective function value
   $$\bar{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$
5. if $\bar{\theta} < \theta$, then $\theta = \bar{\theta}$; a = a+1
6. if $a < ITEM$, back to (2)
7. STOP

5 HEURISTICS TO GET INTEGER FEASIBLE SOLUTION

We could adopt a branch and bound approach, solving a sequence of quadratic programming in the same manner as integer program are solved with a sequence of (continuous) linear program. However, for large problem the computation would be prohibitive with a lot less effort we may obtain an integer feasible solution of the QAP using an initial starting point which has been generated randomly. We use two kinds of heuristics method to get the integer feasible solution of QAP. We call two heuristics method as Forward Exchange Strategy and Backward Exchange Strategy.

5.1 FORWARD EXCHANGE STRATEGY ALGORITHM

1. Input *n* as problem dimension.
2. Read flow matrix [f_{ik}], distance matrix [d_{je}], initial asssignment [x_{ij}].
3. Calculate
   $$\theta = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$
4. Generate $x_i=j$ for $x_{ij}=1$; *i,j*=1, …, *n*
5. *a*=1
6. *b*=1
7. *c*=Gen $x_{i=a}$; Gen $x_{i=a}$=Gen $x_{i=a+b}$; Gen $x_{i=a+b}$=*c*
8. x_{ij}=1 for Gen $x_i=j$, $x_{ij}=0$ for Gen $x_i \neq j$, *i,j*=1,…, *n*
9. Calculate objective function
   $$\bar{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$
10. if $\bar{\theta} < \theta$ then $\theta = \bar{\theta}$ go to (14)
11. c = Gen $x_{i=a+b}$ = Gen$x_{i=a}$; Gen $x_{i=a}$=*c*
12. *b* = *b*+1
13. if $b \le n - a$ go to (7)
14. *a* = *a*+1
15. if $a \le n$ go to (6)
16. STOP
17.

5.2 BACKWARD EXCHANGE STRATEGY ALGORITHM

1. Input *n* as problem dimension
2. Read flow matrix [f_{ik}] , distance matrix [d_{je}], initial assignment [x_{ij}].
3. Calculate $\theta = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$
4. Gen $x_i=j$, for x_{ij}=1 ; i,*j* = 1, …, *n*
5. *a* = *n*
6. *b* = *n*
7. *c*=Gen $x_{i=a}$; Gen $x_{i=a}$= Gen $x_{i=b}$; Gen $x_{i=b}$ =*c*
8. x_{ij}=1 for Gen $x_i=j$, $x_{ij}=0$ for Gen $x_i \neq j$, *i,j*=1,…, *n*
9. Calculate objective function
   $$\bar{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$
10. if $\bar{\theta} < \theta$ then $\theta = \bar{\theta}$ go to (14)
11. c = Gen $x_{i=b}$ = Gen $x_{i=a}$; Gen $x_{i=a}$=*c*
12. *b* = *b-1*
13. if $b \ge a + 1$ go to (7)
14. *a* = *a*-1
15. if *a* > 1 go to (6)
16. STOP

6. COMPUTATIONAL EXPERIENCE

6.1. 30 × 30 PROBLEM

The 30 × 30 QAP problem adopted from Nugent (1968). This is a large scale problem with 900 binary variables. Firstly we use random point strategy to get the initial assignment of the QAP and then we use forward exchange strategy to get the integer feasible solution. The result of initial assignment (30 × 30) using random point strategy can be seen in table 1 below.

TABLE 1
INITIAL ASSIGNMENT USING RANDOM POINT STRATEGY

J =						
i = 1	22	27	30	4	9	7
i = 1	10	15	20	18	23	21
i = 1	26	5	3	8	6	11
i = 1	16	14	19	17	25	28
i = 1	2	13	24	1	12	29
Objektif θ = 7666						

Using this initial assignment as a starting point we use forward exchange strategy to get the solution of QAP 30 $\times$ 30. Tables presented below are the result of every iteration using the forward exchange strategy.

TABLE 2
THE RESULT OF FORWARD EXCHANGE STRATEGY ITERATION 1

J =						
i = 1	30	10	18	5	7	8
i = 7	21	15	25	20	16	27
i = 13	26	6	4	14	9	23
i = 19	22	12	19	19	11	28
i = 25	2	13	24	1	3	17
Objektif θ = 6932						

TABLE 3
ITERATION 2

J =						
i = 1	27	7	24	5	14	25
i = 7	22	15	8	20	16	30
i = 13	13	6	12	10	9	29
i = 19	21	4	19	23	17	28
i = 25	3	26	18	1	2	11
Objektif θ = 6718						

TABLE 4
ITERATION 3

J =						
i = 1	28	14	24	5	25	13
i = 7	22	15	8	20	16	29
i = 13	7	6	12	10	21	30
i = 19	9	4	19	17	23	27
i = 25	3	26	18	1	2	11
Objektif θ = 6572						

TABLE 5
ITERATION 4

J =						
i = 1	28	13	24	5	25	8
i = 7	15	16	20	14	17	21
i = 13	7	6	12	9	29	30
i = 19	10	4	19	22	23	27
i = 25	3	26	18	2	1	11
Objektif θ = 6532						

TABLE 6
ITERATION 5

J =						
i = 1	28	19	24	5	25	20
i = 7	15	16	13	14	17	21
i = 13	8	6	18	9	29	30
i = 19	10	4	1	22	23	27
i = 25	3	26	12	2	7	11
Objektif θ = 6272						

TABLE 7
ITERATION 6

J =						
i = 1	28	19	18	5	25	20
i = 7	15	16	13	14	17	21
i = 13	8	6	30	9	29	24
i = 19	10	4	1	22	23	27
i = 25	3	26	12	2	7	11
Objektif θ = 6172						

The value of objective function for iteration 7 equals to the value of objective function for iteration 6. Therefore the local optimal point has been found, iteration stop. In this case we obtain the result of QAP 30 x 30 with the value of objective function θ = 6172, the final assignment shown in Table 7.

6.2. 42 × 42 PROBLEM

The 42 × 42 QAP problem adopted from Skorin-Kapov (1990). The firstly, we use random point strategy to get an initial assignment. Then we use backward exchange strategy for solving the problem. The result is presented in Table 8 which is the 8th iteration of backward exchange strategy. The value of objective function θ is 16166.

TABLE 8
THE FINAL RESULT OF QAP 42X42 USING BACKWARD EXCHANGE STRATEGY

J =									
i = 1	41	24	39	14	30	36	4	29	13
i= 10	16	1	11	7	10	20	37	2	23
i= 19	17	12	26	35	42	34	22	19	25
i= 28	28	15	31	6	3	21	5	18	38
i= 37	33	40	32	27	8	9			
Objektif θ = 16166									

7. THE COMPARISON OF OBJECTIVE VALUE

The comparison table of objective value shown below.

TABLE 9
THE COMPARISON OF OBJECTIVE VALUE

Dimention	Objective value	Methode
30 x 30	6172	Forward exchange strategy
30 x 30	6186	Modified Branch and Bound (Burkard and Stratmann, 1978)
30 x 30	6378	Algoritma Sub Optimal, Gaschutz and Ahrens (1968)
42 x 42	16166	Backward exchange strategy
42 x 42	16470	Tabu search (Skorin-Kapov, 1990)

7. CONCLUSION

The procedure described here has proved successful in obtaining integer feasible solution to the Quadratic Assignment Problem in relatively short computing times (as can be seen from the number iteration needed).

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