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Far East Journal of Mathematical Sciences (FJMS) 2015 Puspha Publishing House, Allahabad, India http://dx.doi.org/10.17654/FJMSJan2015_113_132 Volume 96, Number 1, 2015, Pages 113-132 ISSN:0972-0871 THE SCRAMBLING INDEX OF PRIMITIVE TWO-COLORED TWO CYCLES WHOSE LENGTHS DIFFER BY 1 Mulyono, Hari Sumardi and Saib Suwilo Department of Mathematics University of Sumatera Utara Jl. Bioteknologi No.1

FMIPA USU Medan 20155, Indonesia Abstract A two-colored digraph is a digraph each of whose arc is colored by red or blue. An _-walk is a walk consisting of _red arcs and _blue arcs. The scrambling index of a two-colored digraph is the smallest positive integer _over all nonnegative integers _and _such that for each pair of vertices _there is a vertex _ such that there exist an _-walk from _ and an _-walk from _.

We study the scrambling index of primitive two-colored digraph consisting of two cycles whose lengths differ by 1. We present a lower bound and an upper bound for the scrambling index for such two-colored digraph. We then show that the lower and the upper bounds are sharp bounds. Receive: September 24, 2014; Accepted: November 24, 2014 2010 Mathematics Subject Classification: 05C20, 05C50.

Keywords and phrases: primitive digraphs, two-colored digraph, two cycles, scrambling index. Communicated by K.K. Azad Introduction Led D be a digraph on _ vertices _. A walk of length _from _ is a sequence of arcs of the from _. A walk is open if _ and is closed otherwise. A _ path is a walk with no repeated vertices, expect possibly _. A cycle is a closed path.

The distance from vertex _, denoted by _, is the length of the shortest _ path. A digraph

D is strongly connected provided for each ordered pair of vertices _ there is a _ walk. By a two cycles we mean a strongly connected digraph consisting of exactly two cycles.

A strongly connected digraph D is primitive provided there is a positive integer _ such that for each ordered pair of vertices _ there exist a _ walk. The smallest of such positive integer _ is the exponent of D. in 2009, Akelbek and Kirkland [1] introduced a new parameter on primitive digraph called a scrambling index.

The scrambling index of a primitive digraph is the smallest positive integer k such that for each pair of vertices _ there exist a vertex _ such that there are a _ walk and a _ walk in D. see [1,2] for earliest discussion on the scrambling index of primitive digraphs. A two-colored digraph _ is a digraph each of whose arcs is colored by either red or blue.

An _-walk in a two-colored digraph _ is a walk consisting of h red arcs and _ blue arcs. An _-walk from _is also denoted by _ walk. For a walk W and b(W) to the number of blue arcs in W. the length of W is _. The vector _is the composition of the walk W. The notions of primitivity and exponent of diagraph have been generalized to that of two-colored digraph [3,4].

A strongly connected two-colored digraph _ is primitive provided there exist nonnegative integers _and _ such that for each pair of ordered vertices _ in _ there is a _ walk in _. The smallest positive integer _ over all such nonnegative integers _and _ is called the esponent of _. For a primitive two-colored digraph on n vertices _, we define the scrambling index of _, denoted by _, to be the smallest positive integer _ over all nonnegative integers _and _ such that for each pair of vertices _ in _ there is a vertex _ with the property that there are a _ walk and a _ walk.

We discuss the scrambling index of primitive two-colored two cycles whose lengths _ and s for some positive integer _. In Section 2, we discuss primitivity of such two-colored two cycles in Section 3, we present a way in setting up a lower bound and an upper bound for scrambling index.

In Section 4, we present result on the scrambling index of two-colored two cycles whose lengths differ by 1 Primitivity Let _ be a strongly connected two-colored digraph and let the set of all cycles in _ be _. We define a cycle matrix of _ to be a 2 by q matrix _. If the rank of M is 1, the content of M is defined to be the greatest common divisor of the determinants of 2 by 2 submatrices of M, otherwise.

A two- colored digraph _ is primitive if and only if the content of M is 1 [3]. Let s and c be integers with _and _. We discuss primitivity of two-colored two cycles on _ vertices _ as shown in Figure 1. Let _ be the cycle of length _ and let _ be the cycle of length s. Notice that _ have c vertices in common. / Figure 1. Two cycles whose lengths differ by 1 Proposition 2.1.

let _ be a strongly connescted primitive two-colored two cycles of lengths _ and s, respectively. The cycle matrix of _ is either of the form _. Proff. The cycle matrix of _is of the form _for some _. Since _ is primitive, _. This implies _. If _, then _. Since _, we conclude that _. Therefore in this case we have _. If _then _. Since _, we conclude that _. This implies _. Hence we now conclude that either _.

We assume without loss of generality that the cycle matrix of _is the matrix _. Hence either _ has two blue arcs or _has only one blue arc. Lower and Upper Bounds In this section, we discuss a way in setting up bounds for the scrambling index of two colored two cycles. Proposition 3.1. Let _ be primitive two-colored two cycles and let _ path that contains a vertex of both cycle.

If for some nonnegative integers h and _ the system _ has nonnegative integer solution, then there is a _ walk. Proff. Let _ be the solution to the system and let _ be a vertex in the path _that lies on both cycle. The walk that starts at _, moves to _ along the path _, and moves _ times around the cycles _, respectively, and back at _ and finally follows the path _to _-walk from _.

For a vertex _in _, the local scrambling index of _ at the vertex _denoted by _, is the smallest positive integer _ over all pairs of nonnegative integer _ such that there are a _ walk and a _ walk. The local scrambling index of vertices _in _, denoted _, is defined to be _ . Hence _. The following lemma is a basis for finding a lower bound for the scrambling index of two-colored two cycles. Lemma 3.2.

Let _ be a primitive two-colored two cycles with cycles matrix M and let _. Let _be any two distinct vertices in _. If _is obtained by an _-walk, then _ And hence _fore some paths _ Proof. Since _, there are integers _ such that _. Since every walk can be decomposed into a path and some cycles, _for some path _from _ and some nonnegative integer vector z. comparing these equations we have _.

Hence _ Thus _ for some path _from _. Similarly, we have _ for some path _from _. If _ is obtained by an _-walk then _ Thus _ for some paths _ Results We begin with the case where _ has only one blue arc. Notice that the blue arc of _ must be of the form _ for some _. Lemma 4.1.

Let _ be a primitive two-colored two cycles with cycles of length _ and s as shown in

Figure 1. If _has a unique blue arc _, for some _, then _ Proof we assume that there are a _ walk and a _walk for some vertex _. Let __ _. We consider two cases depending on the position of the vertex _. Case 1. The vertex _ lies on the _ path. There are two paths _ from _.

They are an _-path and an _-path. Considering the _-path we have _ Considering the _-path we have _. So we choose _. There are two paths _from _. They are an _-path and an _-path. Considering the _-path we have _ Considering the _-path we have _. So we choose _. By Lemma 3.2 we conclude that _ (1) For each vertex _ that lies on the _path. Case 2. The vertex _ lies on the _path. There is a unique path _from _ which is a _-path.

Using this path we have _. There is a unique path _ from _ which is a _-path. Using this path we have _. Since _, by lemma 3.2 we conclude that _ (2) For each vertex _ that lies on the _path or _ path. From (1) and (2), we conclude that _ and hence _. Since _. We next discuss the case where _ has two blue arcs.

We first consider the case where the blue arcs have the same initial vertex and then discuss the case where the blue arcs have different initial vertices. Lemma 4.2. Let _be a primitive two colored two cycles of lengths _ and s as shown in Figure 1. If _ has two blue arcs _ and _, then _. Proof. Suppose there are a _ walk and a _ walk. Define _. We consider three cases. Case 1. The vertex _ lies on the _ path.

There is a unique _ path from _ which is an _-path. Using this path we find that _. There are two paths _, from _. They are an _-path and an _-path. Considering the _-path we have _. Considering the _-path we have _. Thus we conclude that _. Lemma 3.2 implies that _ (3) For each vertex _ that lies on the _ path. Case 2. The vertex _ lies on _ path. There is a unique _ path from _ which is an _-path. Using this path we find that _.

There is a unique path _ from _ which is a _-path. Using this path we have that _. Since _. Lemma 3.2 implies that _ (4) For each vertex _ lies on _ path. Case 3. The vertex _ lies on _ path. There is a unique path _ from _ which is an _-path. Using this path we find that _. There is a unique path from _ which is an _-path. Using this path we find that _. By Lemma 3.2 we have _ We consider _ walk. Since the path _ is an _-path, the solution to the system _ Is _ and _.

This implies there is no _-walk from _. We note that the shortest _ walk that contains at least _ red arcs and at least s blue arcs is an _-walk. This implies _ _. Since _ lies on _. Therefore _ (5) For each vertex _ lies on _ path. From (3),(4) and (5) we conclude that _. Hence _ We now discuss the case where _ has two blue arcs with different initial vertices. Lemma 4.3.

Let _ be a primitive two-colored two cycles with cycles of lengths _ and s as shown in Figure 1. If _has two blue arcs _ and _ for some _ and _, then _. _ Proof. For simplicity, we define _. We consider two case where _. Case 1. _. Suppose there are _ walk and _ walk for some vertex _ in _. Let _ and _. We consider three subcases depending on the position of the vertex _. Subcase 1a. The vertex _ lies on the _ path or _ path.

There is a unique _ path from _ which is a _-path. Using this path we have _. There is a unique path _ from _ which is a _-path. Using this path we have _. Lemma 3.2 implies. _ (6) For each vertex _ that lies on the _ path or _ path. Subcase 1b. The vertex _ lies on _ path. There is a unique path _from _ which is a _-path. Using this path we have _. There is a unique path _ from _ which is a _-path. Using this path, we have _.

From Lemma 3.2, we have _. We consider walk from the vertex _. We note that the path _ from _ is _-path. This implies the solution to the system _ Is _ and _. Since the path _ lies entirely on _, there is no walk from _ that consists of _ red arcs and _ blue arcs. Notice that the shortest walk from _ that contains at least _ red arcs and at least _ blue arcs in the walk with _ red arcs and _ blue arcs. This implies _.

Since _ lies on _, we have _. Therefore _ (7) For each vertex _ lies on the _ path. Subcase 1c. The vertex _ lies on _ path. There is unique path from _ which is a _-path. Considering this path we have _. There is a unique path from _ which is a _-path. Considering this path we have _. From Lemma 3.2, we have _ We consider walk from _. We note that the path _ from _ is a _-path. This implies the solution to the system _ Is _ and _.

Since the path _ lies entirely on the cycle _, there is no walk from _ that consists of _ red arcs and _ blue arcs. Notice that the shortest walk from _ that contains at least _ red arcs and at least _ blue arcs in the walk with _ red arcs and _ blue arcs. This implies _. Since _ lies on _, we have _. Therefore _ (8) For each vertex _ that lies on the _ path.

From (6), (7) and (8) we conclude that _ and hence _ (9) Whenever _ Case 2. _. Suppose there are _ walk and _ walk for some vertex _ in _. Let _ and _. We shall show that _We consider three subcases depending on the position of the vertex _. Subcase 2a. The vertex _ lies on _ path or _ path . There is a unique path _from _ which is a _-path. Using this path we find that _. There is a unique path _ from _ which is a _-path.

Using this path we find that _. From Lemma 3.2, we have _ (10) for each vertex _ lies on _ path or _ path . Subcase 2b. The vertex _ lies on _ path. There is a unique path _from _ which is a _-path. Using this path we find that _. There is a unique path _ from _ which is a _-path. Using this path, we find that _. Since _ lies on _, we have _. From Lemma 3.2, we have _ (11) For each vertex _ lies on _ path. Subcase 2c.

The vertex _ lies on _ path. There is a unique path _from _ which is a _-path. Using this path we find that _. There is a unique path _from _ which is a _-path. Using this path we find that _. Since _ lies on _, we have _. From Lemma 3.2, we have _ (12) For each vertex _ lies on _ path. From (10), (11) and (12), we conclude that _and hence _ (13) Whenever _. Finally, from (9) and (13), we conclude that _ Theorem 4.4.

Let _ be a primitive two-colored two cycles with cycles of lengths _ and s as shown in Figure 1. If _ then _ Proof. From Lemma 4.1, Lemma 4.2 and Lemma 4.3, we conclude that _. We next show that _. We shall show that there exist a vertex _ in _ such that for each vertex _, there is a _-walk from _. We shall show the system _ (14) has a nonnegative integer solution for some path _ from _.

The solution to the system (14) is _ and _ We first consider the case where _ has a unique blue arc. Let _ to be the terminal vertex of the blue arc. Then for any path _we have _. Using this path we have _ and _. Since _ and _ we have _. We next consider the case where _ has two blue arcs and let _. If _, then _ lies on the _ path or _ path. The implies _ and hence _ and _. If _ lies on the _ path or _ path, then _ and _.

We note in this case that _ and _ thus _. For each vertex _, there is a vertex _ such that the system (14) has a nonnegative integer solution for some path _. Proposition 3.1 guarantees that for each vertex _, there is a _-walk from _. Thus _ We note that the bounds given in Theorem 4.4 are sharp bounds as shown in the following corollaries. Corollary 4.5.

Let _ be a primitive two-colored two cycles with cycles of lengths _ and s as shown in Figure 1. If _ has a unique blue arc _, then _ Proof. By 4.1, _. It remains to show that _. We show that for each _, the system _ (15) Has nonnegative integer solution for some path _ from _. The solution to the system (15) is _ and _. If the vertex _ lies on the _ path, then there is an _-path from _. Using this path we have _ and _.

Since _ we have _. Since _ we have _. If the vertex _ lies on the _ path, then there is an _-path from _. Using this path we have _ and _. Since _ we have _. Since _ we have _. If _ lies on the _ path, then there is an _-path from _. Using this path we have _ and _. Since _ lies on the _ path, _. Hence _. Since _ we have _. Therefore for each vertex _, there is a path _from _ such that the system (15) has a nonnegative integer solution. Proposition 3.1

guarantees that each _, there is a _ walk with _ and _. Thus _. Now we conclude that _.

Corollary 4.6. Let _ be a primitive two-colored two cycles with cycles of lengths _ and s as shown in Figure 1. If _ has two blue arcs _ and _ and _, then _ Proof. By Theorem 4.4, _. Note that this is a special case of Lemma 4.3 with _ and _. Case 1 of the proof of Lemma 4.3 guarantees that _. Therefore, _. Theorem 4.7.

Let _ be a primitive two-colored two cycles with cycles of lengths _ and s as shown in Figure 1. If _ then _ Proof. From Lemma 4.1 and Lemma 4.3, we conclude that _. It remains to show that _. We shall show that there exist a vertex _ in _ such that for each vertex _, there is an _-walk from _. It suffices to show that there is a vertex _in _ such that for each _, the system _ (16) Has a nonnegative integer solution for some path _from _.

The solution to the system (16) is _ and _. Asuume _ has only blue arc and let _ to be the vertex _ for some _. Then for each _, _, there is an _-path from _ with _. Using this path we find that _ and _. If _, then there is a (1,0)-path from _. Using this path we find that _. We now assume _ has two blue arcs. Since _, we have _. Therefore, there are two possibilities for the two blue arcs of _.

They are either the arcs _ and _ or the arcs _ and _. If _is a blue arc, we let _ to be _. For each _, there is an _-path from _ to _ with _. Using this path we find that _ and _. We note that there is a (1,0)-path from _. Using this path we find that _. If _is a blue arc, we let _ to be vertex _. For each _, there is an _-path from _ to _ with _. Using this path we find that _ and _. We note that there is a (1,0)-path from _.

Using this path we find that _. Therefore, there is a vertex _ in _ such that for each _, the system (16) has a nonnegative integer solution for some path _. This implies for each _ there exist a vertex _ in _ such that there is an _-walk from _ . hence _. We note that the bounds given in Theorem 4.7 are sharp bounds. From Corollary 4.5 the lower bound is achieved if the two-colored two cycles _ has a unique blu arc _.

The upper bound is achieved if the two-colored two cycles _ has two blue arcs with the same initial vertex _ and _. Lemma 4.2 guarantees that _. Combining this and Theorem 4.7, we have _ Acknowledgement This research is supported by The University of Sumatera Utara under Graduate Program Research Grant No:12/UN5.2.3.1/KEU/SP/2014. References [1] M. Akelbek and S. Kirkland, Coefficients of ergodicity and the scrambling index, Linear Algebra Appl. 430(2009), 1111-1130. [2] M. Akelbek and S.

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