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Knowing Nature Through Mathematical Science

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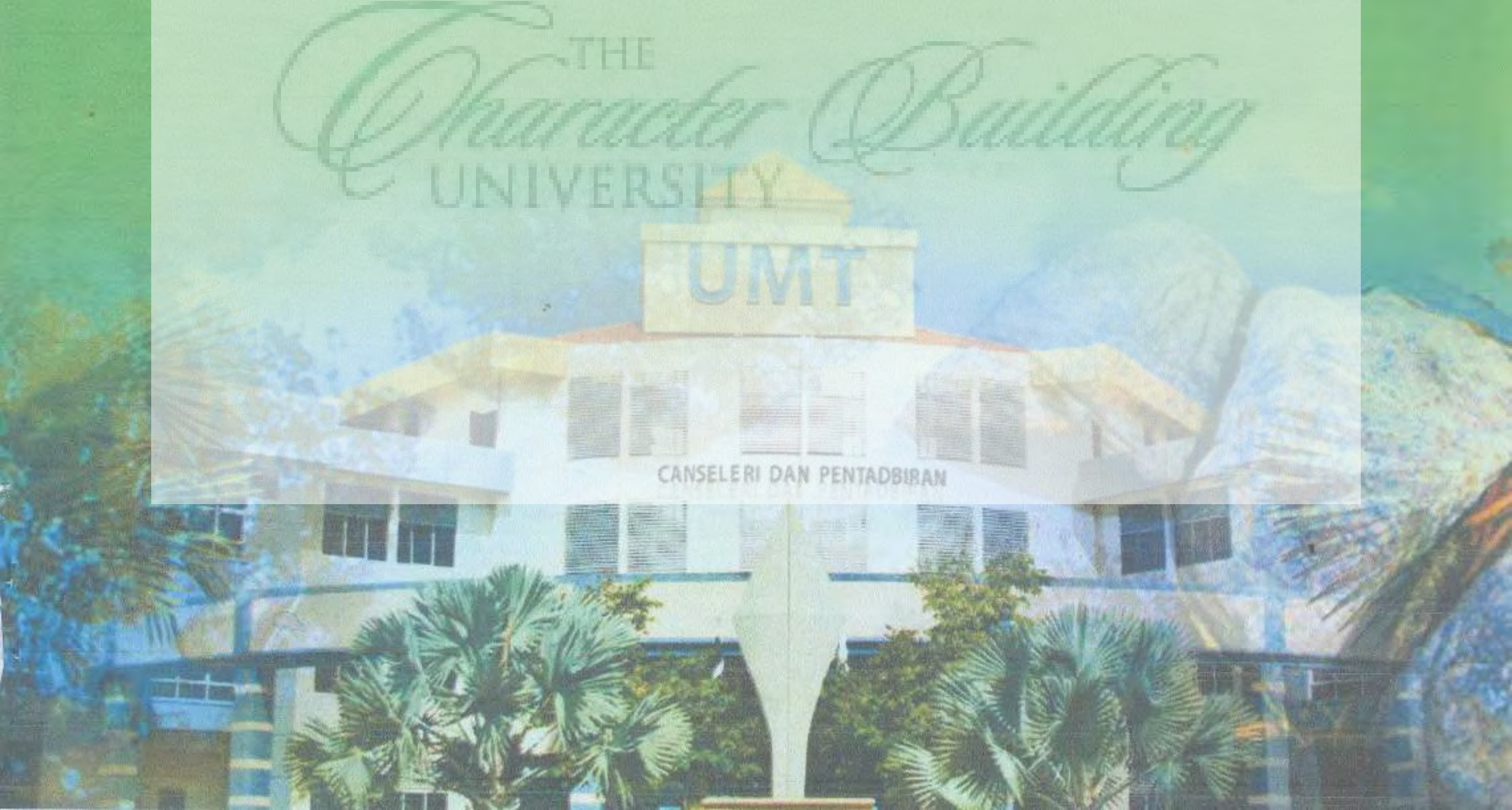

KEMENTERIAN PENDIDIKAN MALAYSIA

UNIVERSITI MALAYSIA TERENGGANU

PERSATUAN SAINS MATEMATIK MALAYSIA
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PROCEEDINGS OF THE 10^{TH} IMT-GT INTERNATIONAL CONFERENCE ON MATHEMATICS, STATISTICS AND ITS APPLICATIONS 2014 (ICMSA2014)

Knowing Nature through Mathematical Sciences

14 -16 October, 2014

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PREFACE

The 10[th] IMT-GT International Conference on Mathematics, Statistics and Its Applications, be held in Kuala Terengganu, Malaysia, on the 14 – 16 October, 2014. This is an annual meeting bringing together mathematician and scientists within the regions of Indonesia, Malaysia and Thailand to discuss and share their findings and experts. With the theme "Knowing Nature through Mathematical Sciences", the scope of the conferences includes various disciplines of mathematics, statistics and computer sciences. All the paper in this proceeding is orally presented in this conference.

For this conference, over 150 of abstracts received by Organizing Committee. On behalf of the conference Committee, the editors would like to express their deepest gratitude to all presenters, contributors/authors and participants of this conference for their overwhelming supports that turns this conference into a big success. While every single effort has been made to ensure consistency of format and layout of the proceedings, the editors assume no responsibility for spelling, grammatical and factual errors. Besides, all opinions expressed in these papers are those from the authors and not the conference organizing committee or the editors. On behalf of the Organizing Committee, we would like to extend our heartfelt thanks to the participants in making this conference a success. We are thankful to the experts who have reviewed the papers which were submitted for this proceeding.

Many grateful to the Universiti Malaysia Terengganu and PERSAMA for the kind of facilities and the financial support in order to makes this conference success. Again we would like to thank the participants.

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Preface

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A MIXED INTEGER NONLINEAR OPTIMIZATION MODEL FOR POSITIONING A PRODUCT IN A MARKET WITH RISK

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Abstract. This is a marketing problem faced by a firm which wishes to position a new brand product in an existing product class. Individuals usually differ in their choice of an object out of an existing set, and they would also differ if asked to specify an ideal object. The aim of the problem considered is to optimally design a new product in order to attract the largest number of consumers. This paper addresses a mixed integer nonlinear programming model to formulate the positioning problem. A direct search approach is proposed to solve the model. A computational experience is presented.

Keywords: Positioning product problem, modeling, MINLP, direct search
PACS: 02.50 Le, 02.60 Pn

INTRODUCTION

There is no product in the world that does not have a position. Product positioning is about visibility and recognition and what product represents for a buyer. In markets where the intensiveness of rivalry and competition are increasing and buyers have a greater intrinsic values become critical. An offering with a clear identity and orientation to needs will not only be purchased, but can warrant a large margin through increased added value.

There is a great number of different definitions of positioning in scientific literature of marketing. The concept of positioning seeks to place a product in a certain respective buyers. Marketers offerings from those of competitors and to create promotions that communicate the desired position. Boone and Kurtz [20], Scientists Etzel, Walker and Stanton [21] refer to a product and to differentiate (position) it in a favorable way from similar products. Ries and Trout [22] distinguish from all other marketing theoretics, stating that positioning is not what is done to a product. Positioning is what you do to the mind of the prospect. The same authors indicate that positioning starts with a product. A piece of merchandise, a service, a company, an institution, or even a person. However, Kotler[23] defines positioning as the act of designing the upies a distinct and valued position in the target consumers mind. Scholars Kotler, Armstrong [23] verify that market positioning is arranging for a product to occupy a clear, distinctive, and desirable place, in the minds of target consumers, relative to competing products. Thus, marketers plan positions that distinguish their products from competing products and give them the greatest strategic advantage in their target markets.

The purpose of positioning is to create a unique and favorable image in the minds of target customers, Bhat [24]. The author Fill [25] states, that positioning, therefore, is the natural conclusion to the sequence of activities that constitute a core part of the marketing strategy. Market segmentation and target marketing are prerequisites to successful positioning.

Product positioning refers to the positioning of a product in a perceptual attribute space such that it closely matches the consumer perception of the various product attributes [18]. For a marketing manager, optimizing a new Product's positioning is a critical and difficult decision. Addressing this issue, [16] developed a framework for identifying optimal new product concepts using joint space models of consumer perceptions and preferences. Joint space analysis entails mapping the locations of existing products and ideal points for each individual (or market segment) use multidimensional scaling (MDS) of consumer perceptions via factor analysis, discriminate analysis or similarity scaling. Using this joint mapping of ideal points and product locations, a manager can model consumers' choices of existing products, predict their responses to new products, and identify optimal new product concepts.

[9] also emphasizes about the important of product positioning in marketing management. [10] discuss about positioning products in which the market has rapid changes in technology and customer preferences. They

propose a mathematical model, in which the model aims to maximize the profit from remanufacturing, given a number of units of end-of-life product.

In the ensuing time period, there have been a number of algorithms developed to identify optimal new product positions from MDS-based maps of consumer perceptions and preferences. Thorough reviews of the MDS-based product positioning literature can be found in [17], hereafter SMS, [6] and [8]. Each step in this evolution was motivated, in part, by attempts to improve the realism of the consumer choice setting. For example, the algorithms that account for a probabilistic choice model tends to provide better solutions, larger share projections, for new product positions [17].

Michalek et al., [26] present a unified methodology for product line optimization that coordinates positioning and design model to achieve realizable firm-level optima. Tinelli et al., [27] propose aa ABC curve method for product positioning optimization. Pharmaceutical companies are adopting various positioning strategies. Kumar and Srivastava [28] presents a model based on conceptual understanding of various positioning typologies with respect to pharmaceutical companies.

In this paper we assume that the consumer first decides his/her budget for buying from a product class. Then the consumer identifies the set of products from the product class that meet his/her budget constraint, evaluate them with the help of a weighted multi-attribute utility model and chooses the product with the highest utility. Therefore we could propose a mixed integer nonlinear programming (MINLP) model to solve the firm's problem of identifying an optimal new product position. The objective is to identify a point in the multi-dimensional attribute space that is closer than the existing product in the product class to the ideal point of as many consumers as possible.

The organization of this paper is as follows. In the next section, we briefly discuss previous research on MDS-based optimal product positioning and building the 'perfect' product. This is followed by a description of the model. The algorithm and results are presented next. We conclude the paper with a discussion of the result.

Optimal Positioning Literature Review

In their review, [16] formalized the process of identifying optimal new product concepts using input from consumers at every stage from defining the market to predicting the success of a new product. Since then, a number of algorithms have been developed for MDS-based product positioning. The early approaches [2, 3, 5] had two limitations in common. First, the search methods for these procedures were dependent on the number of ideal points (individuals or segments) in the joint space. Consequently, as the number of ideal points rose, so did the complexity of the optimization problem. Second, these algorithms were formulated for the single choice problem in which the demand from each ideal point is assumed to be completely captured by the closest product to it. In essence, this model suggests a consumer always chooses the product nearest to their ideal. While the first limitation simply slowed down the convergence to a suitable solution, the second limitation ignored empirical evidence about the nature of consumers' choices in many consumer markets.

It has been shown in studies of panel data (beginning with [12] that consumers often choose probabilistically from a small set of products in the market. One might attribute this behavior to the effects of promotions or availability. However, it has been observed that even if all brands are equally available at no cost, most (53 out of 77) consumers do not choose only their most preferred brand [4]. This indicates that the probabilistic choice behavior may be a product of variety seeking or factors other than environmental effects [14].

In 1987, SMS presented a new product positioning algorithm called PRODSRCH which incorporated a probabilistic model of consumer choice. In their formulation, demand from an ideal point is distributed to a product in inverse proportion its relative distance from the ideal point so long as the product is within the fixed size choice set of the ideal point. Otherwise, the product captures no demand share from that ideal point.[11] address a new methodology for optimal product positioning by considering engineering constraints. The method is based on perceptual mapping and house quality in order to link the consumer perceptual space, and product engineering space.

To illustrate the differences between the single choice model and the probabilistic choice model, we will use the [15] spatial choice model for finite ideal points. This notation will be used throughout the balance of the paper.

- $x_{i,p}$ is the location *i*th ideal point on the *p*th dimension,
- $y_{j,p}$ is the modal perception of the *j*th product on the *p*th dimension,
- $w_{i,p}$ is the relative importance of the *p*th dimension to the *i*th ideal point,
- S_i is the sales potential for ideal point *i*.

The weighted Euclidean distance ($d_{i,j}$) between the ith ideal point and jth product position is given by Eq. (1).

$$d_{i,j} = \left(\sum_{p} w_{i,p}(x_{i,p} - y_{i,p})^2 \right)^{\frac{1}{2}} \tag{1}$$

In the single choice model, the demand captured by product j is S_i if $d_{i,j} < d_{i,J}$ for all $j \neq J$. In the probabilistic choice model, the share of an ideal point's demand captured by a given product j is determined by the size of the choice set (k) and the relative distances of all available products. It is assumed that due to self interest, consumers are more likely to choose products closest to their ideal points [1].

The brand share for product j from the ith ideal point ($\pi_{i,j}$) is based on Eq. (2):

$$\pi_{i,j} = \begin{cases} \dfrac{(1/d_{i,j})}{\sum_{k}(1/d_{i,k})} & \text{for the } k \text{ closest products} \\ 0 & \mathit{otherwise} \end{cases} \tag{2}$$

To determine the demand for product j, the share from the ideal point ($\pi_{i,j}$) is multiplied by the sales potential of the ith ideal point (S_i).

In an extensive simulation comparison, SMS showed that PRODSRCH performs better than the earlier algorithms of [3] and [5] in situations where consumers allocate their demand probabilistically. In the single choice situation, one version of the [5] algorithm performs very well.

Another advantage of PRODSRCH is that it relies on a well tested general purpose non-linear programming algorithm known as QRMNEW [13]. Consequently, the complexity of the problem is determined by the number of dimensions of the search space (product dimensions) rather than the number of ideal points and product positions. For MDS-based product positioning, PRODSRCH is currently considered to be best approach for the single product location problem ([6], p. 132). [7] proposed a Conjoint Analysis approach for solving the positioning of a product problem.

Model Formulation

Let π_B denote the number of existing brands in the market, π_A the number of determinant attributes (dimensions in the MDS joint space), and π_N the number of new products to be introduced.

$Y_j=\{y_{jp}\}$ is the modal perception of the j-th product on the p-th dimension;

$w_i=\{w_{ip}\}$ is the vector of dimension weights for the i-th segment;

$I_i=\{I_{ip}\}$ is the ideal point for the i-th market segment. It is assumed to be *finite*, but need not lie in the region where feasible products might be located;

d_{ij}= the weighted Euclidean distance from the j-th product to the i-th segment's ideal point;

s_i= the i-th segment's demand;

π_{ij}= the share of the i-th segment's demand allocated to the j-th product alternative.

$\pi_{ij}=f\left(d_{ij}^{-1}\right)$

and

$$\sum_{j=1}^{n_B} \pi_{ij} = 1$$

For each $i = 1,2,\ldots,n_M$ before entry,

and

$$\sum_{j=1}^{n_B+n_N} \pi_{ij} = 1$$

For each $i = 1,2,\ldots,n_M$ after entry.

The model posit various functions for the behavior of including probabilistic ones that depend upon distance from ideal point as well as the typical deterministic (first-choice) rule. They assume that the objective is to maximize incremental market share; they also allow for the possibility of cannibalization after introduction. In their notation,

Ψ_i = the set of k closest products before introduction;

x_i = the i-th firm's self-products before introduction;

π_{ij} = likelihoods of purchase before introduction of new products;

$x_n = \{x_{np}\}$ = the n-th new product's position (in the n_A attribute space R_{n_A}); $n = 1,2,\ldots, n_N$;

L = an arbitrarily large number; and

Ψ_i^*, x_i^* and π_{ij}^* are the after-entry (of n_N new products) equivalents of Ψ_i, x_i and π_{ij} respectively.

Then we formulate (and solve) the mixed integer nonlinear programming problem,

$$\text{Maximize } \sum_{i=1}^{n_M} \left(\sum_{j\in x_i} u_i \pi_{ij}^* - \sum_{j\in x_i} \pi_{ij} \right) S_i$$

Subject to

$$d_i^{(k)}(1-u_i) \le \left[\sum_{p=1}^{n_A} \left(I_{ij} - x_p\right)^2 w_{ij} \right]^{\frac{1}{2}}$$

$$< d_i^{(k)} + L(1-u_1),$$

For all $x_n \in R_{n_A}$, and $i \in n_M$, where u_i is zero or one depending on whether (1) or not (0), a self product (existing or new, located at $\{x_p\}$), is among the k closest for the i-th segment.

The Algorithm

The first four sets of Figure 1 partition the full index set , {1, 2, ..., n}, ie $J_B \cup J_S \cup J_L \cup J_U = \{1, 2, \ldots, n\}$ and $J_{\square} \cap J_{\square}$□□□□∅, □ ≠β. The set J_I of indices corresponding to integer variables is assumed to be of small cardinality, and $m + n_S + n_L + n_U = n$.

The approach assumes that the continuous problem is solved, and seeks an integer-feasible solution in the close neighbourhood of the continuous solution. The general philosophy is to leave non-basic integer variables at their respective bounds (and therefore integer valued) and conduct a search in the restricted space of basics, superbasics, and nonbasic continuous variables, $j \notin J_I$.

The algorithm may be broadly summarized as follows:

1. Obtain solution of the continuous relaxation as a nonlinear programming problem.
2. CYCLE1: remove integer variables from the basis by moving a suitable nonbasic away from its bound. The hope is to drive an infeasible integer basic variable to an integer value, and then to pivot it into the superbasic set; the previous nonbasic replacing it in the basis.

Some notation is first needed. We define the required index sets.

Name	Meaning	Cardinality
J_B	set of indices for basic variables	$\mid J_B \mid = m$
J_S	set of indices for superbasic variables	$\mid J_S \mid = n_S$
J_L	set of indices for nonbasic variables at their lower bounds	$\mid J_L \mid = n_L$
J_U	set of indices for nonbasic variables at their upper bounds	$\mid J_U \mid = n_U$
J_I	set of indices for integer variables	$\mid J_I \mid = n_I$

Figure 16 Index sets for extended simplex partition

3. CYCLE2, pass1: adjust integer-infeasible superbasics by fractional steps to reach complete integer-feasibility.
4. CYCLE2, pass2: adjust integer feasible superbasics. This phase aims to conduct a highly-localized neighbourhood search see Scarf [83] to verify local optimality.

In Cycle1, there are several steps.

Step 1. Get row i^* the smallest integer infeasibility, such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

Step 2. Do a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

Step 3. Calculate $\sigma_{i_j} = v_{i^*}^T a_j$

With j corresponds to

$$\min_{j}\left\{\left|\frac{d_j}{\sigma_{ij}}\right|\right\}$$

Calculate the maximum movement of nonbasic j at lower bound and upper bound Otherwise go to next non-integer nonbasic or superbasic j (if available). Eventually the column j^* is to be increased form LB or decreased from UB. If none go to next i^*.

Step 4.

Solve $B\alpha_{j^*} = \alpha_{j^*}$ for α_{j^*}

Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic j^* from its bounds.

Step 6. Exchange basis

Step 7. If row $i^* = \{\emptyset\}$ go to Stage 2, otherwise

Repeat from step 1.

CONCLUSION

Firms work to position and design lines of products that best suit to market and profitability goals. The firms can interpret this imperative by measuring the customer preferences and positioning new products for marketers and maximizing performance under technological constraints. This paper presents a mixed integer nonlinear programming model to describe the positioning of a new product based on multidimensional scaling (MDS) of consumer perceptions.

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