SCIENCE, MATHEMATICS, ENGINEERING AND TECHNOLOGY EDUCATION: CULTURAL CHALLENGES AND OPPORTUNITIES IN A GLOBALISING WORLD

8th 2015

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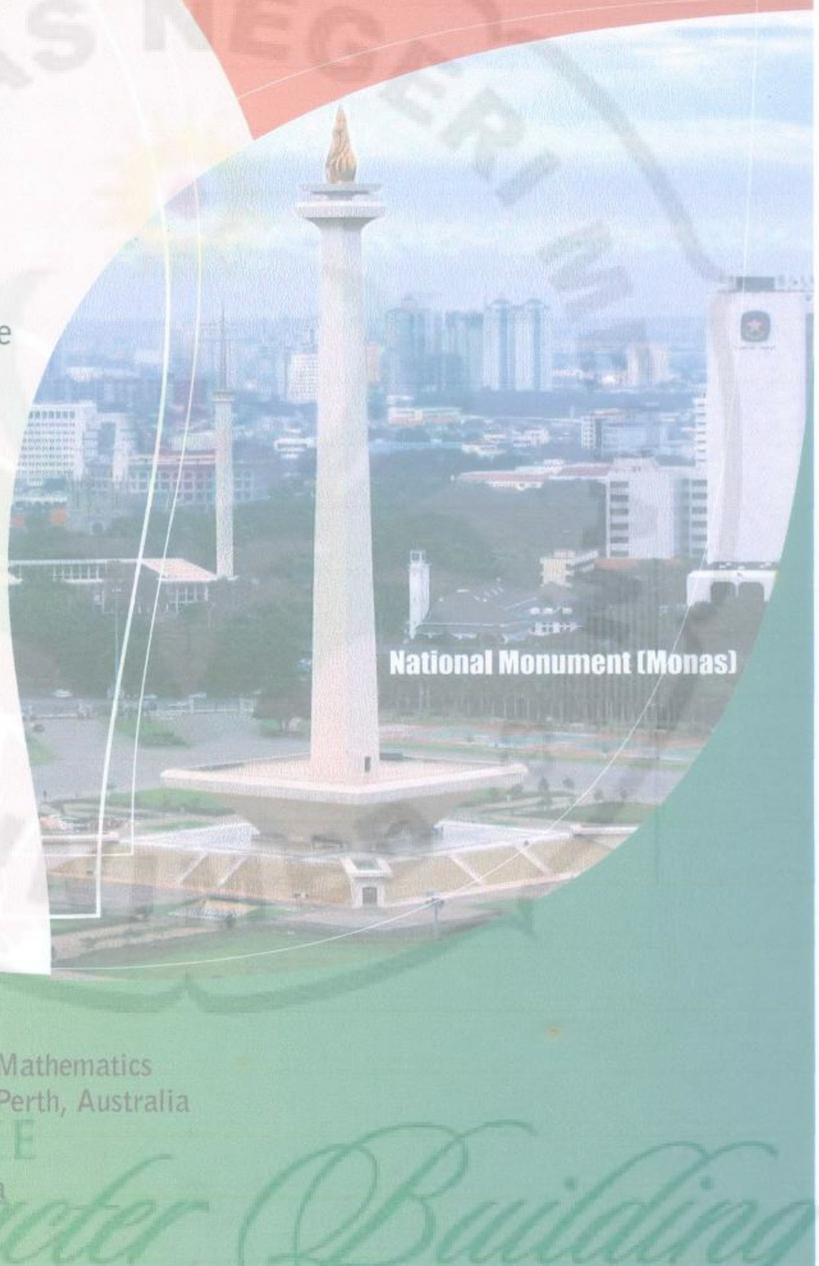
8th International Conference on Science, Mathematics and Technology Education



Proceedings of the
Eighth International Conference on Science, Mathematics, Engineering and Technology Education

Sari Pan Pacific Hotel, Jakarta, Indonesia
21-24 November, 2015

CURTIN UNIVERSITY
AUSTRALIA

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Organisers:

Jointly organised by the Science and Mathematics Education Centre, Curtin University, Perth, Australia

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Science, Mathematics, Engineering and Technology Education – Cultural Challenges and Opportunities in a Globalising World

Proceedings of the Eighth International Conference on Science, Mathematics and Technology Education

Jakarta, Indonesia
21 – 24 November, 2015

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TABLE OF CONTENTS

MATHEMATICAL UNDERSTANDING AND REPRESENTATION ABILITY OF PUBLIC JUNIOR HIGH SCHOOL: PRELIMINARY RESULTS

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Abstract: Indonesia faced the fact that the ability of mathematical problem solving ability of the students is so low. So, like other countries, Indonesia has the program to enhance this ability. The author conducted the research for the purpose of developing mathematical understanding ability (MUA) and mathematical representation ability (MRA) as the basis for problem solving, This paper described the result of a preliminary study to investigate both MUA and MRA of public junior high school (PJHS) grade 8. The population is all of the students of PJHS in Medan (North Sumatera) and Bandung (West Java). The samples are 33 students from PJHS 3 Bandung and 40 students from PJHS 27 Medan city. The techniques used for collecting data are test of MUA and MRA. Data is analysed descriptively. The research results show that the students' achievement in MUA test as well as in MRA test is low.

Keywords: Mathematical Understanding, Mathematical Representation

INTRODUCTION

In order to improve the quality of education, each country seeks to make a good curriculum which is fit with the culture of the country concerned. For example, Singapore sets the five pillars of education to support the fulfilment of the curriculum (Lee $ Tan, 2004), the US published NCTM (2000), which was preceded by three great works of others, i.e., Curriculum and Education Standards for School Mathematics (1989), Professional Standards for School Mathematics (1991), and Assessment Standards for School Mathematics (1995). Indonesia itself has undergone a change of curriculum several times to fit with the times, ranging from Curriculum 1975 to Curriculum 2013 (K-13). Currently, the government urged the schools to return to the Competency-based Curriculum (KTSP, 2006).

In KTSP 2006, the general objective of education includes laying the basis of intelligence, knowledge, personality, character, and skills to live independently and to follow further education. Especially for mathematics, the objectives of giving mathematics in school is that students understand mathematical concepts , describes the relationship between concepts and apply concepts or algorithms in a flexible, accurate, efficient, and precise in problem solving. It implied that the students must be grasp the ability of mathematical understanding, mathematical connection ability, and mathematical problem-solving abilities which is in line with that proposed by NCTM (2000).

The ability of solving mathematical problems is definitely a must since the ultimate goal of learning anything is to enable one to solve problems. On the other hand, a student will be able to solve a mathematical problem if he/she has the capability of mathematical understanding, connection, representation and reasoning abilities. In other words, the ability of understanding, connection, representation and reasoning ability is underpinned problem solving. Conversely, when a student works on math problems or solving problem, he/she was sharpening the problem-solving ability (Minarni, 2015).

The ability of mathematical understanding and representations are the basis of mathematical problem solving. In fact, mathematical problem solving ability of the students of public junior high school (PJHS) in several countries, including Indonesia, was not adequate. The capability of mathematical representation of PJHS students can be seen in TIMSS 2011 (Mullis, 2012). Countries whose students achieve the best score in solving mathematical problem are Korea and the State of Singapore, with score respectively 613, 611. Indonesian students earned a score of 386, far from these two countries.

According to Hiebert and Carpenter (1992), understanding is in terms of the way information is presented and structured. A mathematical idea or procedure or fact is understood if it is part of an internal representation (internal network). More specifically, mathematics is understood if its mental representation is part of network representations. On the other side, representation ability is the ability of the students to represent information or problems in the form that make them easier in understanding that problems.

Understanding ability can be grasped by the students (Mously, 2004). This ability can be developed through five features of meaning-oriented instruction, they are:

1. Broadening the range of mathematical content to give students a sense of the breadth of mathematics and its implications.
2. Emphasising connections between mathematical ideas.
3. Exploring mathematics that is embedded in rich and real life situations.
4. Encouraging students to find multiple solutions and focusing students' attention on links between the solution processes used.
5. Creating multiple representations of ideas (e.g., drawings and physical objects).

In this study, the ability of mathematical understanding and representation are investigated as a goal, a process and as a tool through problem solving activities. Through this activity the students demanded to declare the problem into a form that makes them easier to understand situation of the problem and understand what is required in the problem.

Some aspect of understanding which are proposed by Anderson (2001) will be used to analysed the students' understanding ability. The aspects are included interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. Meanwhile, the following statements adapted from NCTM (2000) are used to analyse mathematical representation ability of the students, these are: (1) Visual representation including charts, tables, drawings and sketches, and diagrams; (2) Nonvisual representation including a numeric representation (number) and mathematical equation or mathematical model.

The achievement of various mathematical abilities of the students primarily depends on learning approach used by the teachers in the classroom (Slavin, 2006; Ronis, 2008). Until now, it must be recognized that the pattern of teaching learning in schools is still dominated by ordinary (classical/conventional) learning approach. Such approach puts teachers in a central position in the classroom, so it is called as teacher-center learning approach, it means that learning is dominated by teachers through lectures and expository method. The teachers rarely trigger the students to analyze in depth about a concept and seldom prompt students to use high logical reasoning as proving a principle. The teachers have no time to give the students the opportunities to present, communicate, and apply math in the context of everyday life because of too many subject mattes included in the curriculum.

The students who are taught in conventional classroom would be difficult and could not move forward whenever faced non-routine questions or complicated problems which cannot be solved in just one step completion. Such learning approach teachers used will not allow the students to gain the ability of problem solving or other high order mathematical thinking skills such as reasoning and representation skill. That's why in CBC 2006, the teachers is provoked to use innovative learning approach that enable the students construct their own knowledge as proposed by constructivism. When the students are able to construct their own knowledge then it can be expected they will be able to solve problems since the process of constructing knowledge required patience, thoroughness, perseverance, curiosity, ability to connect prior knowledge with new knowledge. It is similar to the process of problem solving.

In this research, we investigate whether the teachers know learning approach such as problem-based learning approach (PBL) that emphasizes the importance of the students' involvement in creating or constructing their own knowledge (Arends, 2008), whether the teachers know the innovative learning approach such as RME (Realistic Mathematics Education), Discovery Learning, Open-ended approach and others. If the teachers know such innovative learning approach, do they implement it.

The result of interview showed that the teachers did not want to use the student-centered learning approach because of it is time consuming and difficult to find the students who want to persevere. Thus, even if the teachers are trained and insisted to apply such learning approach, they will go back to use conventional one. The purpose of this research is to reveal whether the teacher used the student-centered learning approach in the classroom, made material-instruction by themselves, made the instrument of the test at the end of the semester (not designed along with material instruction at the beginning of the semester), and whether the students' engagement in learning activity is good enough. But reported here is the main purpose of the research that is the topics related to the ability of understanding and ability of mathematical representations of students.

THEORITICAL FRAMEWORK

Learning with understanding is both essential and possible in school mathematics and supported by learning principle. The argument in favor of meaningful learning in school mathematics was made and supported experimentally as early as the 1930s and has been elaborated since then by many proponents of learning with understanding. It has also been corroborated by the results of many recent studies of varying instructional and theoretical approaches. Schoenfeld (1992) stated that these studies collectively emphasize the importance of having meaning related to learning activities of students of varied ages, backgrounds, and abilities; and reveal the need for more instructional attention to sense-making as part of school mathematics instruction.

In supporting of learning principle, the research suggests that all students can understand and apply important mathematical concepts. Also, this scholarly work emphasizes the merits of students developing conceptual understanding, and stresses the importance of the powerful connections established between procedures and concepts when one practices this kind of learning.

According to Marzano and Kendall (2007), understanding involves two related processes: integrating and symbolizing of knowledge. Integrating involves reducing knowledge down to its key parts. In technical term, integrating is about creating a macrostructure for knowledge usually at a more general level than originally experienced, for example identifies the defining characteristics of a generalization or principle.

The understanding process of symbolizing involves depicting knowledge in some type of non-linguistic or abstract form, such as when asked, the student accurately represents the major aspects of details in non-linguistics or abstract form. For example, to elicit knowledge symbolizing, asked the student to illustrate what they consider to be the important aspect of the equation $y = 2x$ using a graphic representation or a table.

Important statement derived from Hiebert and Carpenter (1992) that define understanding in terms of the way information is presented and structured. A mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, mathematics is understood if its mental representation is part of network representations.

In addition, learning mathematics with understanding involves *making connections* among ideas; these connections are considered to facilitate the transfer of prior knowledge to novel situations. Transfer is essential because most new problems require solution via previously learned strategies; it would be impossible for one to become mathematically competent if each problem required a separate strategy.

Sierpinska (1994) clarified this by putting forward three different ways of looking at understanding. First of all, there is the '*act of understanding*' which is the mental experience associated with linking what is to be understood with the 'basis' for that understanding. Examples: mental representations, mental models, and memories of past experiences. Secondly, 'understanding' which is acquired as a result of the acts of understanding. Thirdly, there are the 'processes of understanding' which involve links being made between acts of understanding through reasoning processes, including developing explanations, learning by example, linking to previous knowledge, linking to figures of speech and carrying out practical and intellectual activities.

Nickerson (1985) examined understanding as an example agreement with experts, being able to see deeper characteristics of a concept, look for specific information in a situation more quickly, being able to represent situations, and envisioning a situation using mental models. He highlighted the importance of knowledge and of relating knowledge: 'The more one know about the subject, the better one understand it. The richer the conceptual context in which one can embed a new fact, the more one can be said to understand the fact.'

Anderson (2001: 70-75) stated that the students are said to understand when they are able to construct meaning from instructional massages, including oral, written, and graphic communication presented to them during lectures, in books, or on computer monitors. Students understand when they build connections between the 'new' knowledge to be gained and their prior knowledge. More specifically, the incoming knowledge is integrated with existing schemas and cognitive frameworks. Since concepts are the building blocks of schemas and frameworks, then conceptual knowledge provides a basis for understanding. Cognitive process in the category of understanding are consisted of interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.

Mathematical representation is a fairly complex cognitive process in learning mathematics, especially when such representation is intended to make the important mathematical concepts more easily understood. It will be more complicated if representation required to present in various types although it can be used by students to help them in developing a more profound understanding and flexible on a concept (Hiebert and Carpenter, 1992; Skemp, 1987).

Carpenter & Lehrer (1999) stated that representation can be classified into two categories, namely internal representation and external representation. Internal representation is everything that exists in the cognitive structure of students, while the external representation can be poured from the internal representation into visual form (picture, table, graph, sketch, symbols) and nonvisual representation such as mathematical equation. Representation in the form of words; can be oral or written. Equations can be categorized as a visual representation because it involves symbols.

According to Meira (2002), the students think through some representation models. Representation can bridge difficulties in understanding math and can make mathematics more attractive and interesting (not rigid and monotonous). Representation helps the students in presenting clearer picture and a better understanding of a concept or idea.

NCTM (2000: 334) stated that mathematical representation will enables the students to create and use representations to organize, record, and communicate mathematical ideas; selecting, implementing, and convert a form of representation to another representation to solve the problems; using representations to model and interpret physical phenomena, social, and mathematical phenomena itself. The following statements are standard mathematical representation process adapted from NCTM (2000): (1) Visual representation including charts, tables, drawings and sketches, and diagrams; (2) nonvisual representation including a numeric representation (number) and mathematical equation or mathematical model.

In line with the Principles and Standards for School Mathematics (NCTM, 2000), the school mathematics curriculum in Indonesia stressed that the teaching program for kindergarten through grade XII should make the students be able to construct a variety of representations and use them in problem solving.

RESEARCH METHODS

The population of the research is Public Junior High School (PJHS) grade 8 in Medan city (North Sumatera) and Bandung city (West Java). Sample is randomly selected. There are two samples included in the study, one from PJHS 27 in Medan, another one is PJHS 3 in Bandung city. The instrument used to measure the ability of mathematical understanding and mathematical representations are two sets of essay tests. A set of mathematical understanding test consists of seven problems. Mathematical representation test consists of six problems. Math materials tested include Pythagorean rule and linear equation of two variables.

Aspects of mathematical understanding used in this research are adapted from Anderson (2001), including:

1. Interpreting: Stating the information in the problem to form a system of linear equations of two variables.
2. Exemplifying: Giving specific examples of the concept of line equation.
3. Classifying: Grouping (classifying) an example of mathematical equation into linear equations of one or two variables (complementary process of giving an example.
4. Summarizing: Proposed a single statement associated to Pythagorean rule to disclose the information presented.
5. Inferring: Find similarities or patterns in a given equation to determine the requested equation.
6. Comparing: Detecting the difference or similarity between two or more lines to determine a requested gradient of the line.
7. Explaining: Constructing and using the causal model of a system.

To measure the ability of a mathematical representation, the researchers adapted the standard process of mathematical representation process contained in NCTM (2000), these are:

1. Change the information on the problem to the Pythagorean equation and use the properties of Pythagoras to solve problems.
2. Represent scenario of real life problem in mathematical equation to reach the solution
3. Create a linear equation of one variable and interpret that equation according to the initial problem.
4. Using gradient formulation and algebraic manipulation to resolve the problem.
5. Represent the information or problem into graph or chart and use it to solve problems.
6. Using the representation of the problem and relate it to the Pythagorean rule to obtain the solutions.

RESULTS AND DISCUSSION

Mathematical Understanding of the Students

This preliminary study did not intend to discuss the difference of the students' achievement in completing test of mathematical understanding, but only wants to know whether the findings are in line with the findings of TIMMS (2012). And, this study wants to reveal that the students who are taught by conventional teaching learning have always had difficulty in completing non-routine problems. For this purpose, we analyse the students' performance in the test. The achievement of Mathematical Understanding Ability (MUA) of The Students at PJHS 3 Bandung and PJHS 27 Medan is presented in Table 1.

Table 1. Mean and standard deviation of The Students' MUA

	School	N	Mean of MUA Score	Std. Deviation	Std. Error Mean
MUA	PJHS 3	33	11,18	6,267	1,091
	PJHS 27	40	5,40	2,285	,361

MUA Ideal Score Mean = 28

As an example of the student performance in MUA problem is presented below. The purpose of this problem is to elicit the student ability in giving example as well as contra-example of line concept.

Problem 2

Write down the equation of two parallel line and two line orthogonal to each other. Give your explanation.

Answer:

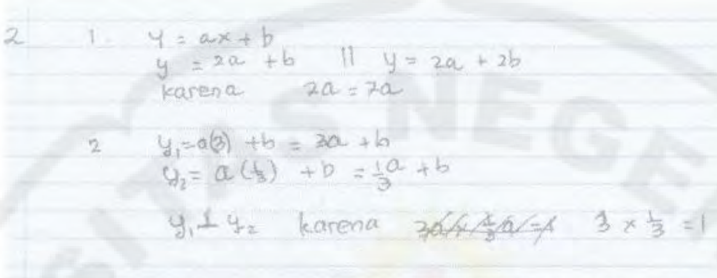

Figure 1. Performance of Problem 2 from Student A at PJHS 27

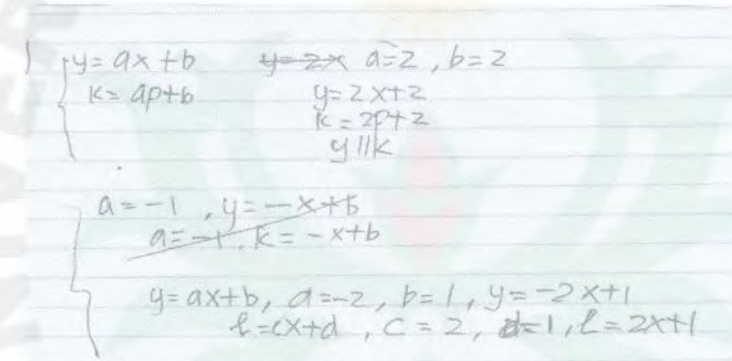

Figure 2. Performance of Problem 2 from Student B at PJHS 3

Problem 2 can be categorized as an open-ended problem, so there will be a lot of different solution. Open-ended problems potentially develop students' skills in the retrieval of knowledge from their cognitive structure. This skill will support the ability to provide examples, which is one of seven indicators of understanding ability. The solution provided by the student at PJHS 27 (Figure 1) indicated that this student has not understood yet the concept of line equation. In particular, he did not know the difference between variables and coefficients precisely so that he mistakenly substitutes the value of the variable. Actually, he should substitute a value of 2 to coefficient of variable X, not substituted it into the variable X.

Solutions provided by one of the students in PJHS 3 (see Figure 2) shows that he has better understanding of the concept of straight line. In connection with the instruction to give the example of two parallel lines, this student gives the correct instance. This shows that he understands the concept of two parallel lines. On the other hand, with regard to the command to write two perpendicular lines, he gives wrong examples. This means that he does not fully understand the rule of two perpendicular lines. Many students provide a solution similar to the solution shown in Figure 2. In short, the students lack of understanding of the properties of two lines equation are probably because of this concept is not store firmly in their cognitive structure as stated by Hiebert & Carpenter (1992).

The students are not only fluent in giving examples of a concept but they also lack in other aspects of understanding ability, such as interpreting, classifying, inferring, comparing, summarizing, and explaining both the problems and their solutions. Analysis based on the solutions given by the students for the test of MUA are summarized as follows: more than 50% of the students at PJHS 27 Medan get score less than 50 in MUA test. While, 30% of the students at PJHS 3 Bandung achieve score above 50%.

The statements supported by the data presented in Table 2 that the achievement of students at PJHS 3 and students at PJHS 27 are equally below 50 %. These findings support the findings of TIMSS (2012), that is mathematical achievement of the students are so low. Interviews and observations reveal the teachers at both schools are still applying the conventional teaching approach, while the students' prior knowledge is not too bad. Thus, it can be presumed that teaching approach used by the teachers is responsible for the students' low ability in completing mathematical problems.

Mathematical Representation of the Students

Mean and standard error of mathematical representation ability (MRA) of the students at PJHS 3 Bandung and PJHS 27 Medan are presented in Table 2.

Table 2. Mean and standard deviation of The Students' MRA

School	N	Mean of MRA Score	%of MRA Mean	Std. Deviation	Std. Error Mean
PJHS 3	33	7,79	32,46	5,260	,916
PJHS 27	40	6,93	24,75	2,683	,424

MRA ideal mean score = 24

Table 2 presents the achievement of students at PJHS 3 and PJHS 27 in the MRA test. Overall, the average achievement of the two classes from the two school do not differ much, which is about 32,46% for PJHS 3 and 24,75% for PJHS 27. The achievement of MRA test is the resume of the students' performance. Figure 3 and 4 show examples of the students' performance in a matter that requires the ability to create charts.

Problem 5

A pair of line are presented below, represent them in chart. Decide which pair is orthogonal and give your reason.

a. $2y - 4x - 4 = 0$ dan $y = 3x - 3$
b. $3y + x - 9 = 0$ dan $y = 3x + 1$

Answer:

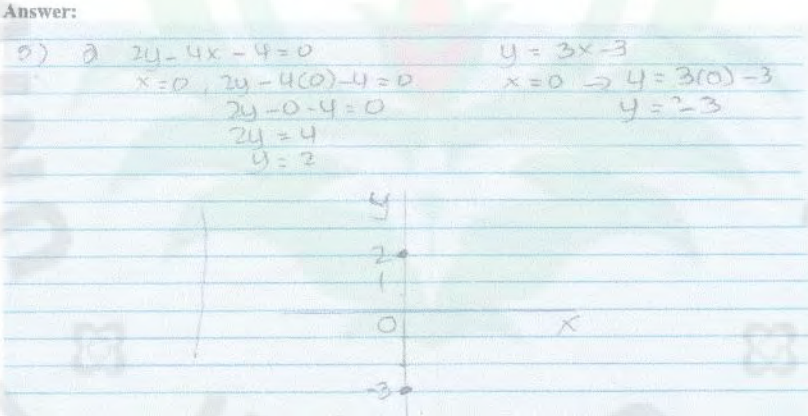

Figure 3. Performance of Problem 5 from Student C at PJHS 27

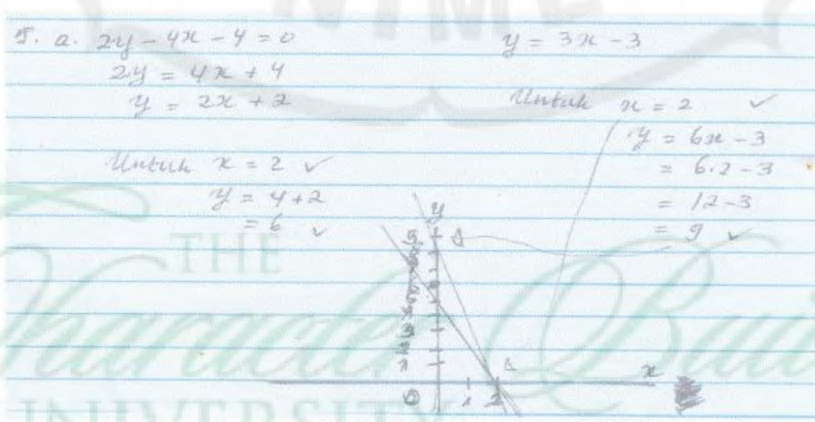

Figure 4. Performance of Problem 5 from Student D at PJHS 3

It can be seen in Figure 3 that this student has not understand the concept of linear equation. There is misconception here, he decided to set zero for variable x, but he did not take any value for variable y, so he could not get a pair of ordered number to enable him in creating chart. Meanwhile, from Figure 4 we can see that this student has the ability to create chart for equation $y = 3x - 3$ and $y = 3x + 1$ even though it's not true. He has difficulty in creating chart for equation $2y - 4x - 4 = 0$ and $3y + x - 9 = 0$. In general, these equation can be stated in the form $ax + by + c = 0$. Actually, he should find two coordinate from these equations to help him in creating chart for an equation. It seems hard enough for the students to create chart for linear equation $ax + by + c = 0$.

Other results of the research show that some students have the ability to create a table to help them solving the problem, but only a few of them has the ability to turn a problem into the form of pictures, graphics, or a mathematical equation that will make it easier to obtain a true solution.

Few students both at PJHS 27 and PJHS 3 can exactly represent line equation into chart, but overall, the students ability in creating chart are so poor. In representing information into chart, more than 50% of the students at both school get score less than 50%. While, the students' score in representing information (problem) into mathematical equation at both school is less than 30%, the average of MRA score are respectively 7,79 or 32,46% for the students at PJHS 3 and 6,93 or 24,75% for the students at PJHS 27.

Results of this research also show that the cause of the inability of students in solving mathematical representations including the weakness of recalling the existing knowledge in cognitive structure that impact on the weakness in transferring knowledge into new situation or problem. That Solution for problem 5 that is proposed by the number of the students indicated that they could not recall knowledge about creating chart. Marzano & Kendall (2007) stated that the students will not able to recall knowledge if the knowledge is not store in strong connection with the existing knowledge in the cognitive structure.

The teachers at PJHS 3 Bandung city and PJHS 27 Medan city know that learning approach such as problem-based learning (PBL), discovery learning, and realistic mathematics education (RME) emphasize the importance of the students' involvement in creating or constructing knowledge. Through such approach, at least, the students' mathematical understanding and representation ability can be achieved since by implementing one of these approaches the students are encouraged to be actively discuss and solve real-life problems. But, the teachers still used conventional teaching learning approach because of it is time consume and required them to be patient whenever confronted with slow learner students.

The ability of mathematical understanding of the PJHS students is low at all, less than 50 %. Provisional estimates, it is because of learning factor. At both schools where the initial research was conducted, conventional learning is still used. In this study, conventional learning is referred to teacher-centered learning, it means that it is the teachers who transfer the knowledge to the students, give some problems and solution, and then students are asked to solve similar problems that are resolved by the teacher. Through this learning, the teacher never presented non-routine problems, no in-depth discussion about mathematical concept/idea between students and teachers. In accordance with the opinion of Ronis (2008), this kind of teaching learning cannot make students achieve problem-solving abilities.

The results of interview and observation with the students who have finished their work in solving the test of mathematical understanding showed that the difficulty the students faced in completing the given problems is because they have forgotten the material being tested. This means, the knowledge obtained by the students are not deeply embedded in their cognitive structure, and according to Marzano & Kendall (2007) and Anderson (2001), it can be because of low ability of understanding. More precisely, Marzano & Kendall (2007) argues that only if students understand the knowledge being studied, then the knowledge will be firmly entrenched in the cognitive structure of students.

Related to the ability of mathematical representation, the data showed that the students' performance in test of mathematical representation is not much different from their performance in test of mathematical understanding ability. Mathematical representation ability is the ability to change/translating the information or problem proposed to other forms that are different from the original form but has the same meaning with original problem. For example, the students insisted to change the information or scenario of real-life problem into picture/graphics/charts/table. Actually, the ability of representation can be controlled by the student if he has the ability in mathematical understanding, because, according to Hiebert & Carpenter (1999), the ability of external representation represents internal representation. If the students' external representation about a concept is firm, then this suggests that the concept contained in the student's cognitive structure is also firm. When the external and internal representation well-meaning then knowledge has been embedded with a powerful and has established a good network in the cognitive structure of students.

CONCLUSION

Based on the research results it can be concluded that the 8^{th} grade students' achievement in mathematical understanding test is categorized as low. The 8^{th} grade students' achievement in mathematical representation test is also categorized as low.

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