A combined strategy for solving quadratic assignment problem

Cite as: AIP Conference Proceedings **1867**, 020006 (2017); https://doi.org/10.1063/1.4994409
Published Online: 01 August 2017

Faiz Ahyaningsih

a Combined Strategy for Solving Quadratic AssignmentProblem

Faiz Ahyaningsih

Department of Mathematic, State University of Medan (UNIMED), Jln Willem Iskandar Psr V Medan Estate, Medan, 20220, Indonesia

faizahyaningsih@yahoo.com

Abstract. The quadratic assignment problem is a combinatorial problem of deciding the placement of facilities in specified locations in such a way as to minimize a nonconvex objective function expressed in terms of flow between facilities, and distance between location. Due to the non-convexity nature of the problem, therefore to get a 'good' starting point is necessary in order to obtain a better optimal solution. In this paper we propose a combined strategy (random point strategy to get initial starting point and then use forward exchange strategy and backward exchange strategy to get 'optimal' solution). As a computational experience we've solved the problem of Esc 16b, Esc 16c and Esc 16h from QAPLIB. Finally, we present a comparative study between Combined Strategy and Data –Guided Lexisearch Algorithm. The computational study shows the effectiveness of our proposed combined strategy.

Keywords : Combined Strategy, Quadratic Assigment Problem, Random Point Strategy

1. INTRODUCTION

The Quadratic assignment problem (QAP) is a simplest form concerned with locating facilities on locations, such that total transportation costs are minimized. The transportation cost incurred by locating two facilities is proportional both to the flow of transportation between the facilities and to the distance between their locations. In 1957 QAP was introduced by Koopmann and Beckmann as a mathematical model for the location of a set of indivisible economic activities. The QAP however is not only nonlinear but also not unimodal. As a consequence this problem class has attracted active investigation by numerous researchers. An heuristic yielding seems more appropriate than attempting an optimal solution, and many of the papers aim to achieve improvements for initial assignment. The paper by Ahmed (2011), provides a good review for recent advances in QAP.

One of the oldest applications of the QAP is the assignment of specialized rooms in a building (Elshafei, 1977). In this case, a_{ij} is the flow of people that must go from service i to service j and b_{ij} is the time for going from room i to room j. A more recent application is the assignment of gates to airplanes in an airport; in this case, a_{ij} is the number of passengers going from airplane i to airplane j and b_{kl} is the walking distance between gate k and gate l.

Finally, Hadley *et. al.* (1992) propose a new lower bound, Laporte and Mercure (1988) apply QAP in turbine runner balancing, and the fact, the problems such as travelling salesman or linear ordering can be formulated as a special QAPs.

2. LITERATURE REVIEW

The quadratic assignment problem was one of the first problems solved by metaheuristics methods first conceived in the 1980's. Burkard and Rendl (1984) proposed a simulated annealing procedure that was able to find

International Conference on Mathematics: Pure, Applied and Computation
AIP Conf. Proc. 1867, 020006-1–020006-6; doi: 10.1063/1.4994409
Published by AIP Publishing. 978-0-7354-1547-8/$30.00

much better solutions than all the previously designed heuristic methods. Six years later, Connolly (1990) proposed an improved annealing scheme. His method is easy to set up, since the user has to select the number of iterations; and all other parameters are automatically computed. At around the same time, Skorin-Kapov (1990) proposed a tabu search. Then, Taillard (1991) proposed a more robust tabu search, with fewer parameters and running *n* times faster than the previous implementation. Even though Taillard's method was proposed over 25 years ago, it remains one of the most efficient for some problem instances. The other tabu search have been proposed, such as the reactive tabu search of Battiti and Tecchiolli (1994), the comparison of tabu search techniques and simulated annealing. Battiti and Tecchiolli (1994), and the star-shape diversification approach of Sondergeld and Voss (1996). One year later T. James *et. al.* (2007), addressed a multi-start tabu search and diversification strategies for the quadratic assignment problem.

Genetic algorithms have also been proposed, for instance by Tate and Smith (1995), but hybrid approaches, such as paper of Fleurent and Ferland (1994), Ahuja *et. al.* (2000), Drezner (2002), are more efficient. Another heuristic, GRASP (greedy randomized adaptive search procedure) was proposed by Li *et. al.* (1994). An ant colony system approaches by Taillard (1998) , Gambardella *et. al*., (1999), Stutzle and Hoos (1999) have been proposed, as well as a scatter search by Cung *et. al.* (1997). Some of these methods have been compared in Taillard (1995). It shows that the efficiency of these methods strongly depends on the problem instance type to which they are applied.

Recently, Ahmed addressed a Data –Guided Lexisearch Algorithm (DGLSA) for solving quadratic assignment problem, that provides a good review for recent advances in QAP, but just only for the instances of size 12 the algorithm is found to be better, for the large sized instances the lexisearch algorithm is not found to be suitable. Also, for some small sized instances, for example esc16b of size 16, the algorithm could not prove optimality of the solution (2014).

In combined strategy starting point plays a vital role in reducing search space, hence, reduce the computational time. Also, by using random point strategy before applying forward exchange strategy and backward exchange strategy, pre-processing of data can reduce the computational effort significantly.

3. THE QUADRATIC ASSIGNMENT PROBLEM

This is combinatorial problem of deciding the placement of facilities in specified locations in such a way as to minimize a quadratic objective function. Consider the problem of locating *n* facilities in *n* given locations. If the flow f_{ik} between each pair of facility *i* and facility *k* and the unit transportation cost (or distance) d_{jl} between locations *j* and *l* are known, then the problem is defined to be

minimize $$\Phi = \tfrac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$

subject to
$$\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1,\dots,n$$
$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1,\dots,n$$
$$0 \le x_{ij} \le 1$$
$$x_{ij} \quad \text{integer}$$

Matrices [f_{ik}] and [d_{jl}] are assumed to be symmetric. The assignment variable x_{ij} has a value 1 if facility *i* is at location *j*, and is zero otherwise. The constraints reflect the fact that each location can be assigned to only one facility, and each facility can be assigned to only one location.

Generally the QAP is a non-convex problem so any solution obtained will necessarily be a local optimum and not a global optimum.

4. A COMBINED STRATEGY

4.1. Strategy to get Initial Starting Point

Since the quadratic form may be non-convex, the chances of obtaining a good integer feasible solution are considerably enhanced by paying some attention to the starting point for the search procedure.

In this case we create a computer program that would generate a random assignment. Then we calculate the value of objective function using this random assignment. In order to get a good initial starting point it is neccessary to input an arbitrary value of objective function θ enough big say θ=10000000, so its value always greater than $\bar{\theta}$, at last from the numbers of iteration $\theta\, min$ will be reached.

The steps used to get a random initial starting point are written as follows :

1. Read θ= 10000000, matrix [f_{ik}] dan [d_{jl}], n = problem dimension and *ITER* as the number of iteration, a = 1
2. Generate random permutation, x_i= randperm (n); for every facility i, i = 1, 2, …, n.
3. $x_{ij}=1$ for gen $x_i=j$; $x_{ij}=0$ for generate $x_i \neq j$; i,j = 1,2,..,n
4. Calculate the objective function value
$$\bar{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$
5. if $\bar{\theta} < \theta$, then $\theta = \bar{\theta}$; $a = a + 1$
6. if $a < ITER$, back to (2)
7. STOP

4.2. Forward Exchange Strategy

We could adopt a branch and bound approach, solving a sequence of quadratic programming in the same manner as integer program are solved with a sequence of (continuous) linear program. However, for large problem the computation would be prohibitive with a lot less effort we may obtain an integer feasible solution of the QAP using an initial starting point which has been generated randomly.

We use random point strategy to get an initial assignment, then combine by heuristic method, namely forward exchange strategy and backward exchange strategy to get the integer feasible solution of QAP.

Algorithm for Forward Exchange strategy :

1. Input n as problem dimension.
2. Read flow matrix [f_{ik}], distance matrix [d_{je}], initial asssignment [x_{ij}]
3. Calculate $\theta = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$
4. Generate $x_i=j$ for $x_{ij}=1$; $i,j=1, \ldots, n$
5. a=1
6. b=1
7. c=Gen $x_{i=a}$; Gen $x_{i=a}$=Gen $x_{i=a+b}$; Gen $x_{i=a+b}=c$
8. $x_{ij}=1$ for Gen $x_i=j$, $x_{ij}=0$ for Gen $x_i \neq j$, $i,j=1,\ldots, n$
9. Calculate objective function
$$\bar{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$
10. if $\bar{\theta} < \theta$ then $\theta = \bar{\theta}$ go to (14)
11. Gen $x_{i=a+b}$ = Gen$x_{i=a}$; Gen $x_{i=a}=c$
12. $b = b+1$

13. if $b \le n - a$ go to (7)
14. $a = a+1$
15. if $a \le n$ go to (6)
16. STOP

4.3. Backward Exchange Strategy

Algorithm for Backward Exchange Strategy :

1. Input n as problem dimension
2. Read flow matrix [f_{ik}], distance matrix [d_{je},], initial assignment X = [x_{ij}.]
3. Calculate $\theta = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$
4. Gen $x_i=j$, for $x_{ij}=1$; i,j = 1, …, n
5. $a = n$
6. $b = n$
7. c=Gen $x_{i=a}$; Gen $x_{i=a}$= Gen $x_{i=b}$; Gen $x_{i=b}$ $=c$
8. $x_{ij}=1$ for Gen $x_i=j$, $x_{ij}=0$ for Gen $x_i \neq j$, $i,j=1,\ldots, n$
9. Calculate objective function

$$\overline{\theta} = \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{n} f_{ik} d_{jl} x_{ij} x_{kl}$$

10. if $\overline{\theta} < \theta$ then $\theta = \overline{\theta}$ go to (14)
11. Gen $x_{i=b}$ = Gen $x_{i=a}$; Gen $x_{i=a}=c$
12. $b = b-1$
13. if $b \ge a + 1$ go to (7)
14. $a = a-1$
15. if $a > 1$ go to (6)
16. STOP

5. COMPUTATIONAL EXPERIENCE

To run the program we use laptop with processor Intel (R) core (TM) i3-3217U, CPU 1.80 GHz RAM 4.00 GB.

5.1. 16 x 16 Problem

The 16 × 16 QAP is adopted from Escermann (1990). This is a large scale problem with 256 binary variables. We try the number of iteration is 500, no other number of iteration, because by using this number of iteration we have found the optimal solution according to the QAPLIB, Burkard *et. al.* (1997)

The result can be seen in table 1.

TABLE 1. The Search Table for Esc16

Instance	Number of Iteration	Objective Value	Permutation	Running Time (Second)
Esc 16b	500	292 (OPT)	2 3 16 9 7 13 1 5 15 11 14 4 12 8 10 6	371.65
Esc 16c	500	160 (OPT)	5 16 15 8 7 6 11 3 4 9 2 10 12 14 1 13	270.09
Esc 16h	500	996 (OPT)	16 15 8 5 14 9 10 6 1 2 13 4 3 11 12 7	326.96

The comparison between Combined Strategy (CS) and Data–Guided Lexisearch Algorithm (DGLSA) by Ahmed (2014) shown in table 2 :

TABLE 2. The Comparison Between DGLSA and CS

Instance	BKV	DGLSA			CS		
		BSV	Error (%)	Running Time (Second)	BSV	Error (%)	Running Time (Second)
Esc 16b	292	292	0.00	14400.00	292	0.00	371.654879
Esc 16c	160	160	0.00	2984.60	160	0.00	270.088915
Esc 16h	996	996	0.00	1298.50	996	0.00	326.956387

BKV = Best known value (from QAPLIB)
BSV = Best solution value
Error = { (BSV-BKV)/BKV } x 100%

TABLE 3. The Comparison of Machine

Methods	Machine
DGLSA	PC Pentium IV, Speed 3 GHz, 448 MB RAM, WINDOWS XP
CS	Laptop Intel (R) core i3-3217U, CPU 1.80 GHz, 4 GB RAM

DGLSA = Data–Guided Lexisearch Algorithm
CS = Combined Strategy

6. CONCLUSION

The combined strategy can solve problem instance Esc 16b, Esc 16c, and Esc 16h from QAPLIB.

The combined strategy (CS) is much more better than Data–Guided Lexisearch Algorithm (DGLSA). It was shown from the running time.

Based on the speed, machine of DGLSA is faster than CS (3 GHz vs 1.80 GHz).

Based on our experiment for problem instances Esc 16b, Esc 16c, and Esc 16h, optimum value reached at iteration 500.

The combined strategy was proved successful in obtaining integer feasible solution to the problem instances Esc 16b, Esc 16c, and Esc 16h in relatively short computing time.

7. REFERENCES

1. Z. H. Ahmed, International Journal of Operational Research **12**, 20–33 (2011).
2. A.N.Elshafei, Operational Research Quaterly **28**, 167-179 (1977).
3. S. W. Hadley, F. Rendl and H. Wolkowicz, Mathematics of Operation Research **17**, 727-739 (1992).
4. G.Laporte and H.Mercure, EJOR **35**, 378-381 (1998).
5. R.E.Burkard, and F.Rendl, European Journal of Operational Research **17**, 169-174 (1984).
6. D.T.Connoly, European Journal of Operational Research **46**, 93-100 (1990).
7. J.Skorin-Kapov, ORSA Journal on Computing **2**, 33-45 (1990).
8. E.D.Taillard, Parallel Computing **17**, 443-455 (1991).
9. R.Battiti and G.Tecchiolli, ORSA Journal on Computing **6**, 126-140 (1994); *ibid.*, Computers and Mathematics with Applications **28**, 1-8 (1994).
10. L.Sondergeld and S.Vob, "A star-shaped diversification approach in Tabu search ", in: "Meta-heuristics: Theory and Applications", I.H.Osman and J.P.Kelly (eds.), (Kluwer academic Publishers, 1996), p. 489-502.
11. T. James, C. Rego, and F. Glover, Technical Report, Virginia Tech, Blacksburg, VA (2007).
12. D.E. Tate and A.E.Smith, Computers and Operations Research **1**, 855-865 (1995).
13. C.Fleurent and J.Ferland, DIMACS Series in Math. Theoritical Computer Science **16**, 190-206 (1994).
14. R. K. Ahuja, J.B.Orlin, and A.Tiwari, Computers and Operations Research **27**,917-934 (2000).
15. Z. Drezner, INFORMS Journal on Computing (**2002**).

16. Y.Li, P.M.Pardalos, and M.G.C.Resende, “A randomized adaptive search procedure for the quadratic assignment problem”, in “Quadratic Assignment and Related Problems”, P.Pardalos and H. Wolcowicz (eds.), DIMACS series in Discrete mathematics and Theoritical Computer Science, **16**, 237-261 (1994).
17. E.D.Taillard, “FANT:Fast ant system “, Technical report IDSIA-46-98, IDSIA, Lugano (1998).
18. L.M.Gambardella, E.D.Taillard and M.Dorigo, Journal of the Operational Society **50**, 167-176 (1999).
19. T.Stutzle and H..Hoos, “the Max-Min ant system and local search for combinatorial optimization problems: Towards adaptive Tools for Global Optimization” :, in : “Meta-heuristics: Advances and Trends in Local Search Paradigms for Optimization”, S.Voss, S.Martello, I.H.Osman, C.Roucairol (eds.), Kluwer academic Publishers, **313-329** (1999).
20. V.-D, Cung, T.Mautor, P.Michelon, and A. Tavares, “A scatter search based approach for the quadratic assignment problem”, in proceedings of the IEEE International Conference on Evolutionary Computation and Evolutionary Programming (ICEC’97),Indianapolis, (IEEE,1997), pp. 165-170.
21. Z. H. Ahmed, Indian Journal of Science and Technology, Vol **7**(4), 480-490 (2014).
22. B. Eschermann and H. J. Wunderlich, “Optimized Synthesis of Self-Testable Finite State Machines”, in 20th International Symposium on Fault-Tolerant, (1990).
23. R. E. Burkard, Karisch SE, Rendl F. QAPLIB - a quadratic assignment problem library. J Global Optim. **10**(4), 391–403 (1997).