
**ABSTRACT**

This paper presents a Bayesian approach to find the Bayesian model for the point forecast of ARMA model under normal-gamma prior assumption with quadratic loss function in the form of mathematical expression.

The conditional posterior predictive density is obtained from the combination of the posterior under normal-gamma prior with the conditional predictive density. The marginal conditional posterior predictive density is obtained by integrating the conditional posterior predictive density, whereas the point forecast is derived from the marginal conditional posterior predictive density. Furthermore, the forecasting model is applied to inflation data and compare to traditional method.

The results show that the Bayesian forecasting is better than the traditional forecasting. Keywords: ARMA Model, Bayes Theorem, Inflation, Normal-gamma Prior JEL Classifications: C13, C15, C22 1. INTRODUCTION Bayes theorem calculates the posterior distribution as proportion to the product of a prior distribution and the likelihood function.

The prior distribution is a probability model describing the knowledge about the parameters before observing the currently by the available data. Main idea of Bayesian forecasting is the predictive distribution of the future given the fast data follows directly from the joint probabilistic model. The predictive distribution is derived from the
sampling predictive density, weighted by the posterior distribution (Bijak, 2010).

This is to and (2015) the problem Bayesian for ARMA under prior. papers to research Amry Fan and Yao (2008), Kleibergen and Hoek (1996), and Uturbey (2006) also discussed the Bayesian analysis for ARMA model. This paper focuses to find the mathematical expression of the Bayes estimator prior with loss and compare traditional method. 2.

MATERIALS AND METHODS The materials in this paper are some theories in mathematics and integration, and the univariate student’s t-distribution and inflation data. The method is study of literatures by applying the Bayesian analysis under normal-gamma prior assumption. ARMA (p, q) model (Liu, 1995) is defined by: yy ee ti ti tj j j q i p = + + + + + + = ? ? ? ? ? ? (1) Where {et} i i d normal random variables with et ~ N(0, t -1), t>0 and unknown, f i and ? j are parameters.

The Bayes theorem (Ramachandran and Tsokos, 2009) stated as: p(y|x ) a p(x|y ) py (y ) (2) Where p(y|x ) is posterior distribution, p(x|y ) is likelihood function and py(y ) is prior distribution. A quantity, is to a distribution n degrees of freedom with mode µ and scale parameter t>0 if it has the probability density function (Pole et al., 1994): pX n n n n x n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n n
Posterior Distribution According to Broemeling and Shaarawy's suggestion (1988), the normal-gamma prior of parameters $\theta$ and $t$ is:

$$\frac{\Gamma\left(\alpha_p\right)}{\Gamma\left(\alpha_t\right)} \frac{\Gamma\left(\beta_p\right)}{\Gamma\left(\beta_t\right)} \frac{\Gamma\left(\gamma_p\right)}{\Gamma\left(\gamma_t\right)} \frac{\Gamma\left(\delta_p\right)}{\Gamma\left(\delta_t\right)} \quad \text{and} \quad \Gamma\left(\alpha_p\right) \frac{\Gamma\left(\beta_p\right)}{\Gamma\left(\gamma_p\right)} \frac{\Gamma\left(\delta_p\right)}{\Gamma\left(\delta_t\right)}$$

By applying the Bayes theorem to equation (10) and (11), the posterior distribution of $\theta$ and $t$ - n is: $p(t, \theta | y) \propto L(\theta) p(\theta) p(t | \theta) p(y | t, \theta)$

$$L(\theta) = \exp \left\{ -\frac{1}{2} \sum_{j=1}^{n} \left( \frac{(y_j - \mu_j)^2}{\sigma_j^2} \right) \right\}$$

$$p(\theta) = \prod_{i=1}^{n} \left( \frac{1}{\sigma_i} \right)^{\alpha_p} \exp \left\{ -\frac{1}{2} \sum_{i=1}^{n} \left( \frac{\theta_i^2}{\sigma_i^2} \right) \right\}$$

$$p(t | \theta) = \prod_{i=1}^{n} \left( \frac{1}{\beta_i} \right)^{\beta_p} \exp \left\{ -\frac{1}{2} \sum_{i=1}^{n} \left( \frac{t_i^2}{\beta_i} \right) \right\}$$

$$p(y | t, \theta) = \prod_{i=1}^{n} \left( \frac{1}{\gamma_i} \right)^{\gamma_p} \exp \left\{ -\frac{1}{2} \sum_{i=1}^{n} \left( \frac{y_i^2}{\gamma_i} \right) \right\}$$

$$p(t, \theta | y) \propto \left( \frac{1}{\sigma_i} \right)^{\alpha_p} \exp \left\{ -\frac{1}{2} \sum_{i=1}^{n} \left( \frac{\theta_i^2}{\sigma_i^2} \right) \right\} \prod_{i=1}^{n} \left( \frac{1}{\beta_i} \right)^{\beta_p} \exp \left\{ -\frac{1}{2} \sum_{i=1}^{n} \left( \frac{t_i^2}{\beta_i} \right) \right\} \prod_{i=1}^{n} \left( \frac{1}{\gamma_i} \right)^{\gamma_p} \exp \left\{ -\frac{1}{2} \sum_{i=1}^{n} \left( \frac{y_i^2}{\gamma_i} \right) \right\}$$

Conditional Posterior Predictive Density Based on $y_e y_t t y_t j q_j i p = - - - - = = ? ? ? ? 1$
Marginal Conditional Posterior Predictive

The marginal conditional posterior predictive density of $Y_{n+k}$ can be obtained by integrating the conditional posterior predictive density in equation (15); density in equation (15): $f_{Y_{n+k}}(y_{n+k}) = \int f_{Y_{n+k}}(y_{n+k} | \theta) f(\theta) d\theta$, where $f(\theta)$ is the prior distribution of the parameter $\theta$. In the case of the normal-gamma prior, the conditional posterior predictive density of $Y_{n+k}$ is given by:

$$f_{Y_{n+k}}(y_{n+k} | \theta) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left( -\frac{(y_{n+k} - \theta)^2}{2\sigma^2} \right)$$

where $\sigma^2$ is the variance of the normal distribution. The marginal density is then obtained by integrating over the prior distribution of $\theta$.

$\sigma^2$ can be calculated using the method of moments or Bayesian analysis.
By applying the formula of Gamma distribution from the equation (16) can be obtained: 

\[ f(y) = \frac{y^{k-1}e^{-y/\theta}}{\Gamma(k)\theta^k} \]

4. APPLICATION

The results of point forecast are applied to a set of time series data that been by ARMA using based on data from 1 to 192.

**Point Forecast**

For quadratic loss function, the point forecast of \( Y_{n+k} \) is the posterior mean of the marginal conditional posterior predictive, that is:

\[ E(Y_{n+k}) = \frac{\int y f(y|x_{n+k}) dy}{\int f(y|x_{n+k}) dy} \]

The results of point forecast are applied to a set of time series data that been by ARMA using based on data from 1 to 192. 4.1.
Data, Stationarity, Identification, and Model Selection

Data 204 and monthly in from January 2000 to December 2016 is displayed in Figure 1. Plot of ACF in Figure 2 in the form of damped sine wave, indicates that the time series data is stationary. Plot ACF in Figure 2 is disconnected after lag plot PACF Figure 3 is disconnected after second lag, these indicate that the appropriate model for data ARMA(2,1).

The of and value is presented as Table 1. The smallest AIC value in Table 1 is 331.79 on ARMA(1,1) model, it means the suitable model for the data is ARMA(1,1) model. In Yt, its model is written: Yt = 8530606=0.3335 Yt -1 + et (18) 4.2. Comparison to Traditional Method The comparison of point forecast between Bayesian forecasting in equation equation (17) with traditional forecasting in equation (18) is presented in the Table 2. Columns 2 through 4 containing the factual data, result of Bayesian forecasting, and result of traditional forecasting.

The comparison of forecast accuracy between Bayesian method and traditional method is presented in the Table 3. Rows 2 through 5 containing the RMSE, MAE, MAPE and U-Statistics. The results show that the forecast accuracy value of the Bayesian method is smaller than the traditional method, so in this case it is concluded that the forecast accuracy for the Bayesian forecasting is better than the traditional forecasting.

The comparison of descriptive statistics between the Bayesian method and the traditional method is presented in the Table 4. Columns 2 through mean, third quartile (Q3), maximum (Max), and standard deviation for factual 192 data the of forecasting for the 12 steps ahead, and 192 factual data and the result of traditional forecasting for the 12 steps ahead.

Plot of factual data, Bayesian and traditional forecasting for the 12 steps ahead is displayed in Figure 4, shows that the plot of factual data is more varied to the plot of Bayesian than the traditional forecasting. 5. CONCLUSION of the point forecast for Bayesian forecasting under normal- gamma prior. The conditional posterior predictive density Table 3: Comparison of forecast accuracy Bayesian Traditional RMSE 0.12476883 0.2545024 MAE 0.09569079 0.1962556 MAPE 47.6807038 81.4006722 U-statistics 0.16897685 0.4092601 Table 4: Comparison of descriptive statistics Data Min.

Q1 Median Mean Q3 Max SD Factual 1–204 140 1.14 0.5 0.55 0.9 3.3 0.589 Factual 1–192, Bayesian 193–204 140 1.13 0.5 0.55 0.9 3.3 0.590 Factual 1–192 Trad. 193–204 140 1.12 0.4 0.54 0.9 3.3 0.588 International Journal of Economics and Financial Issues | Vol 8 • Issue 5 • 2018 102 is obtained by multiplying the normal-gamma prior with the conditional predictive density.
The marginal conditional posterior predictive density is obtained by integrating the conditional posterior predictive density to parameters, whereas the point forecast is derived based on the mean of marginal conditional posterior predictive density that has the univariate The computational results show that the forecast accuracy value of Bayesian forecasting is smaller than the traditional forecasting, while the values of descriptive statistics show that the Bayesian forecasting is closer to the factual data than the traditional forecasting, it indicates that the Bayesian forecasting is better than the traditional forecasting. REFERENCES Amry, Z.


INTERNET SOURCES:

-----------------------------------------------------------------------------------
<1% - http://ccsenet.org/journal/index.php/ijef
1% - http://repo.uum.edu.my/21950/1/IJEFI%206%20S7%202016%20329%20332.pdf